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Private Schools and Residential Choices: Accessibility, Mobility, and Welfare*

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Abstract

Private schools free households from a strict link between residential location decisions and the tax-school quality bundles they consume. In order to study the impact of private schools on educational outcomes, we develop a general equilibrium model that simultaneously incorporates locational choice built on access and locational choice built on tax-school quality attributes of jurisdictions. We conclude that private school choice enhances the welfare of all households—both those attending private schools and those attending public schools—while also working to reduce the amount of housing and school segregation in equilibrium. Investigation of alternative school policies indicates that greater choice, including using targeted school vouchers, can improve welfare and achievement. Finally, we demonstrate how the fiscal burden arising from some households paying less taxes than they consume in public services varies significantly with the structure of school choice options.

KEYWORDS: Tiebout model, sorting, school choice, urban location model, school district consolidation, vouchers

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1 Introduction

Residential location decisions in the United States depend significantly on the school options that are available. Private schools play an important but under-appreciated role in these decisions by breaking the strict link between school choice and residential location. They tend to lessen both residential and school segregation and to improve the welfare of those remaining in public schools. These impacts and their interaction with governmental finance policy only become apparent, however, with an understanding of the general equilibrium outcomes of household residential choices.

In this paper, we focus on the joint location and school choice problem facing the household. Extensive empirical and theoretical work has been done on both parts of this problem, but the separation of the two has distorted the analysis and has precluded understanding the role of private schools. The most stripped down locational choice model that captures the empirical reality of heterogeneous communities with varying fiscal burdens requires combining Tiebout choice of local public schools with locational choice built on employment access. In this, households make choices over both tax rates and housing quality in addition to access to employment. But even the simplest models combining the separate parts quickly become complicated because of the multiple margins of adjustment. This model exceeds our ability to obtain analytical solutions, leading us to employ computational models to solve for key general equilibrium outcomes of policy changes.

Introduction of private schools leads to interesting insights about both locational patterns and the impacts of various school finance policies. The existence of a private school option where school quality does not depend on location weakens the force of Tiebout sorting, and this in turn results in the poor and disadvantaged households being less isolated than they would be when private schools are unavailable. On the other hand, a variety of policies that restrict choice – largely in pursuit of increased equity in schooling – have surprising impacts when the general equilibrium nature of the schooling and location problem is considered. Policy makers frequently appear to act as if believing that school finance and policy changes lead to little reaction in terms of residential location, thus allowing them to focus on just the partial equilibrium impacts of a policy change. A set of frequently discussed options are simulated within our locational model, yielding some unexpected results. For example, when communities cannot differ in their spending, households still sort according to tastes and school quality differs across neighborhoods. In the most basic simulations, the quality at every school deteriorates because of the restrictions on spending (coupled with the tax-quality preferences of households).

Our results demonstrate that restricting choice and access can worsen inequality and overall welfare, so we explore increasing the poor's ability to choose instead. We study, in particular, the effects of two different voucher programs that

reinforce and increase the ability of households to choose their schools. The findings confirm standard public finance results: such redistributive programs should be financed by the state rather than districts and targeting provides more favorable results.

The existing theoretical-computational analysis on the interaction between public and private schools and on related policy issues mostly relies on the Tiebout (1956) framework, focusing on the implications of local public goods and services in residential decisions (see, for instance, de Bartolome (1990), Fernandez and Rogerson (1996), Nechyba (1999, 2000)). The complete stratification predicted by the pure Tiebout framework is, however, much stronger than that observed in reality (Pack and Pack (1977,1978), Davidoff (2005)), suggesting the need for alternative approaches in modeling and more research on policy recommendations. This pure Tiebout approach also overlooks another essential determinant of residential choices, the trade off between accessibility and space, as emphasized by urban location theory.[1] The accessibility-space tradeoff has a different set of implications on the relation between residential choices and land demands of households.

Our paper adopts the approach developed in de Bartolome and Ross (2003) and Hanushek and Yilmaz (2007) that explore merging the Tiebout and urban residential locational frameworks. This "hybrid" approach predicts residential patterns and community heterogeneity that are more realistic than found in either model, and it appears to be the simplest starting point for an exploration of the impacts of choice on equilibrium outcomes.[2] Our work differs from these two papers in a number of ways: We introduce private schools, which alter the link between neighborhood choice and school choice. These alternatives change residential choice incentives and provide more realistic responses to school-finance policy changes. We also introduce policies that increase private school access for low income households and study their consequences. Unlike de Bartolome and Ross (2003), we allow households to choose their housing quality (measured by lot sizes). This allows us to capture an essential but neglected component of the problem, the fiscal burden caused by households that reside in smaller/cheaper housing units in neighborhoods with higher public good provision levels. One of the sorting implications of the Hanushek and Yilmaz (2007) model with only public schools is that school quality is higher in districts with higher spending. Our analysis here reveals how the link between

[1] See Straszheim (1987) and Fujita (1999) for a review of urban literature.

[2] Nechyba (2000,2003) facilitates similar community heterogeneity without modelling space but by imposing a fixed heterogenous housing stock in each neighborhood. This fixes neighborhood populations, and restricts neighborhood compositions and tax base to a great extent. In addition to its spatial features, our approach does not impose any restrictions on lot size choices or neighborhood sizes and populations. Therefore it can be used to address a larger class of problems, especially those relating to changes in land demand, land regulations, and tax bases.

spending and quality at public schools is significantly modified in the presence of private schools because of changes in the peer groups in the public schools.

We start with a formal description of the model and its calibration. From the baseline model we can then analyze how more or less school choice affects the outcomes in terms of household welfare and the distribution of school quality. The options we consider include barring private schools, restricting local variation in spending, and introducing both locally financed and state financed vouchers (which, in this framework, can be interpreted as similar to a charter school).[3] The results indicate that supporting more choice for the poor through targeted vouchers yields better outcomes than efforts that attempt to bring about more equality through restrictions on choice of the wealthy.

2 The Model

We focus on how the existence of a private school option affects the demand for public school quality, and within that context on how different governmental policies for financing schools alter the quality and composition of schools. The analysis relies on a choice model where households differ in income and tastes for education and optimize in terms of housing quality and location along with the bundle of taxes and school quality. Importantly, understanding the special nature of private schools requires extension of the standard Tiebout school choice analysis. The typical model of demand for local public goods follows the Tiebout framework where the only thing that households care about is the local public good (here schools). While this pure Tiebout model might be a useful simplification for some problems, it is very problematic in considering the special nature of private schools.

The standard Tiebout model leads to complete stratification of households – something that conflicts with empirical reality. The source of this problem is the assumption that households care only about the local public good. A quite different perspective comes from urban locational models that emphasize the importance of employment access and the demand for land, at the exclusion of any consideration of local public goods or of local amenities. By combining public-private school choice in a Tiebout economy with Alonso's (1964) basic land use framework, we

[3]The term voucher usually applies to an option to attend a private school. Charter schools are public schools that operate independently from a school district and that generally cannot choose among students if there is excess demand for the school. Instead with excess demand charter schools must generally use a lottery to select students. The use of a lottery has provided one method for assessing the performance of charter schools; see, for example, Abdulkadiroglu et al. (2009). A final choice option that is prevalent in school districts is magnet schools, where the public district offers a specialized school and generally can choose among applicants if there is excess demand. Magnet schools specialize in such things as the arts, college prep, or specific vocational areas.

can characterize the trade-offs households make between housing and access on the one hand and school quality on the other. The equilibrium locational patterns show a heterogeneity of households with jurisdictions that better matches empirical outcomes than either model by itself. More importantly for this analysis, it allows us to understand how a private schooling option that is independent of residential location can affect the optimal locations and tax choices of households.

The conventional monocentric urban model has all employment at the center of a circular central city with a surrounding suburban ring. This structure is convenient for some analyses because the radial symmetry of the area permits simply looking along any ray from the CBD to determine transportation costs, demand for land, and the distribution of households. It is, however, problematic when attempting to look at our school choice problem, because there is no way to trade access for the public good bundle. In equilibrium, each distinct household type would locate in a specific ring around the CBD and would consume the same bundle of taxes and schooling. If there were more household types than jurisdictions, there would be multiple household types within one or more jurisdictions, but there would not be heterogeneity in the sense that representatives of each household type were found dispersed among the available jurisdictions.

We employ a distinctive variant of a monocentric city employment model. All households work in the Central Business District (CBD hereafter) and choose residential lots in the surrounding area. However, instead of the common circular city, we consider a city made up of two side-by-side hemispheres. A straight line that goes through the CBD (e.g., a river) divides the city into two jurisdictions, East (e) and West (w). This stylized geography provides the simplest way to see the implications of the access-public goods tradeoffs and to consider how adding private schooling alters the equilibrium. While many areas might have different geographical structures, many also differ from the conventional circular city. As Rose (1989) points out, half of the 40 most populous metropolitan areas were bound by the Pacific Ocean, Atlantic Ocean, the Gulf of Mexico, or the Great Lakes. Further, with the decline of central cities and the growth of suburbs, much of the relevant competition in access and schools is among different suburbs and might be characterized by the bifurcated city that we employ.

Each jurisdiction provides its residents a local public good, education, for which there is also a *private alternative*, which is financed by tuition revenues. This private alternative breaks the link between neighborhood choice and school choice. The provision level (quality) of public education in a district is endogenous and depends on both the composition of the households and local property tax revenues, where tax rates of which are determined by majority voting. The jurisdictions may differ in their tax and expenditure policies, and agents are allowed to move across jurisdictions, creating a form of competition à la Tiebout (1956). The distance of

home to the workplace matters because of pecuniary and time costs of commuting to work, thus creating the conventional rent gradients found in the standard residential location model.

The following sections describe the formal model with more precision. The key parameters used in the subsequent calibration of the model and analysis of policy impacts are also discussed.

2.1 Households

Households that work at the CBD choose residential lots around it, characterized by the neighborhood (East or West), distance to the city center, and lot size. The neighborhoods in general will differ in the quality of education-property tax rate packages they offer, as discussed in detail below.[4] Each household has one school-age child who can attend the public school in the neighborhood of residence or the private school if admitted there.[5] A member of every household supplies labor to earn an exogenously determined hourly wage W. For simplicity, we consider two discrete skill categories and two different levels of preferences for education. Households differ according to the wages they earn, as well as in their preferences for education. We name the higher income types *skilled* workers (earning W_S), and the lower income types *unskilled* workers (earning $W_U < W_S$). Both household types exhibit variation in their tastes for the local public good, education. Differentiating each as *high valuation* (H) and *low valuation* (L) gives us four household types in total. Let T denote the household type set: $T = \{SH, SL, US, UL\}$.

Households value consumption of a composite consumer good z, land consumption (the size of the residential lot) $s > 0$, leisure $l \in [0, 24]$, and quality of education q at the school the offspring attends. The preferences of a household can be represented by the utility function: $U(\alpha_i, \eta_i; q, s, z, l) = q^{\alpha_i} s^{\eta_i} z^{\gamma} l^{\delta}$ where

[4] In reality, neighborhoods may differ in other amenities that households care about. Although such differences would not remove the incentives we study, they may change some of the magnitudes of the effects.

[5] Many households do not have any children currently in schools. For our purposes, these households can be thought of as a subset of households with low demand for schooling. At the same time, their behavior may be more complex – involving previously schooled kids with moving inertia, grandkids in schools, or recognition that good schools can be capitalized into housing values (e.g., see the review in Black and Machin (2011)). These complications do not, however, have obvious interactions with the impacts of choice and private schooling. While changing some of the magnitudes of effects, they seem unlikely to reverse the qualitative results below.

$\alpha_i + \eta_i + \gamma + \delta = 1$.[6] The α parameter captures the taste heterogeneity for education and can take on two values $\alpha_i \in \{\alpha_H, \alpha_L\}$ such that $\alpha_H > \alpha_L$.

The city has a dense radial transportation system. The further one lives away from CBD, the higher commuting costs he/she will face. In particular, if a household lives r miles away from CBD, the cost of daily roundtrip commute will be ar dollars (pecuniary cost, $a > 0$) and br hours (time cost, $b > 0$), which converts to $bW_i r$ dollars given the opportunity cost of time. We normalize the price of the composite consumption good to one and denote the unit rent of land r miles away from CBD by $R(r)$.

Households pay a property tax on the value of land. Let τ denote the property tax rate as a proportion of daily rent.[7] The budget constraint of a household can be written as:

$$z + (1+\tau)R(r)s + W_i l + p = Y(r) = 24W_i - (a + bW_i)r. \tag{1}$$

for $W_i \in \{W_S, W_U\}$. The p term represents the household's educational expenditures and is zero if the child attends a public school (and thus pays no tuition). The RHS of the equation is household income net of transportation costs.[8]

[6]While a Cobb-Douglas utility function with an income elasticity of one is fairly common, Glaeser, Kahn, and Rappaport (2008) conclude that the income elasticity of land is much lower – in the range of 0.2-0.5. This leads them to look at the role of public transportation in determining the location of poor households in the central city, instead of the demand for land by higher income families. Adding a choice of transportation mode is beyond this analysis, but it is also unlikely to affect our overall results because of the structure of our cities (as hemispheres with the poor living near the CBD in both). Further, from the U.S. Statistical Abstract, the share of shelter in total expenditures is quite constant across income groups and consistent with our calibrations. Shares are 21 percent for households in less than $70,000 income, 19.2 percent for incomes $70,000-100,000, and 19.5 percent for incomes above $100,000 (U.S. Bureau of the Census, 2008).

[7]The conversion can be done as follows: The total annual rent is 365*R*s , and the property taxes paid in a year amount to τ*365*R*S. With an annual interest rate r, the property value (the present value of the perpetual rent stream) is (365*R*s)/r. The annual property tax rate is then the ratio of annual tax paid τ*365*R*S to the property value, i.e., τ*r.

[8]The number of schools in a typical city exceeds the number of employment centers, so the average distance to a school is considerably less than the average distance to the downtown area. Also, travel within the downtown area is assumed to be very quick and inexpensive compared to the average daily commute, allowing us to ignore non-commute transportation costs and travel within the CBD.

Given the market rent curves $\{R_e(r), R_w(r)\}$ in the two jurisdictions, the quality-tax packages $\{(q_e, \tau_e), (q_w, \tau_w)\}$ for each jurisdiction, and the quality of education and prices for private school $\{q_{pr}, p\}$, the household solves the problem:[9]

$$\max_{s,z,l,j\in\{e,w\},q\in\{q_j,q_{pr}\}} U(\alpha_i,\eta_i;q,s,z,l) = q^{\alpha_i} s^{\eta_i} z^{\gamma} l^{\delta} \quad (2)$$
$$s.t.\ \ z + (1+\tau_j)R_j(r)s + W_i l + p = Y_i(r)$$

where $i \in T = \{SH, SL, UH, UL\}$.

2.2 Market Rent Curves and Allocation of Land

Land is owned by absentee landlords and auctioned off to the highest bidder.[10] The reservation price of the landlord, R_a, is determined by an alternative use of land, such as agriculture, and is independent of the location. For a given utility level $\bar{u}$ we can find the maximum rent a household is willing to pay per unit of land and optimal lot size choice r miles away from CBD by solving the problem $\Psi(r,u,q,\tau) = \max_{s,z,l} \left\{ (Y - z - Wl - p)/((1+\tau)s) \middle| U(\alpha,\eta;q,s,z,l) = \bar{u} \right\}$ to obtain the *bid rent function*:

$$\Psi(r,\bar{u},q_j,\tau_j) = \frac{k_i^{1/\eta_i}}{(1+\tau_j)W_i^{\delta/\eta_i}} \; q_j^{\alpha_i/\eta_i} (Y_i(r) - p)^{(\eta_i+\gamma+\delta)/\eta_i} \bar{u}^{-1/\eta_i} \quad (3)$$

and the *bid-max lot size* function:

$$s(r,\bar{u},q_j,\tau_j) = \frac{\eta_i}{(\eta_i+\gamma+\delta)(1+\tau_j)} \; \frac{Y_i(r) - p}{\Psi(r,\bar{u},q_j,\tau_j)} \quad (4)$$

where $k_i = \frac{\eta_i^{\eta_i}\gamma^{\gamma}\delta^{\delta}}{(\eta_i+\gamma+\delta)^{\eta_i+\gamma+\delta}}$, $i \in T$, and $j \in \{e,w\}$.[11] At an auction for a particular location r^*, the winner will be the type with the highest bid rent curve at that location. Given the four types of households in the model, and given that each type has a choice over private and public school, in each jurisdiction we can plot eight bid

[9] For convenience, we abuse the notation and use $W_{SH} = W_{SL} = W_S$, $W_{UH} = W_{UL} = W_U$, $\alpha_{SH} = \alpha_{UH} = \alpha_H$, $\alpha_{SL} = \alpha_{UL} = \alpha_L$, etc.

[10] This assumption eliminates any investment motive for home ownership (based on expected capital gains). Again, attempting to incorporate such motives would be unlikely to change the qualitative general equilibrium results.

[11] For derivations and a detailed discussion of the properties of these bid-rent functions see Hanushek and Yilmaz (2007).

rent curves.[12] The *equilibrium rent curve* $R_j(r)$ is the upper envelope of the bid rent curves of different household types and the agricultural rent R_a. Because all bid rent curves are convex and decreasing, the equilibrium rent curve $R_j(r)$ will be decreasing up to a distance r^*_{jf}, the *fringe distance*, and will stay constant from that point on. Households with steeper bid rent curves will locate closer to the CBD.[13] Higher income increases the demand for land consumption (our measure of housing quality) and attracts households further away from CBD, but it also increases the opportunity cost of commuting time. Private school tuition is equivalent to a decrease in income.

Our city is a *closed city* with the population given exogenously. This allows us to pin down the population density at the equilibrium. Some more notation is necessary to clarify this: Let $L(r)$ represent the land density r miles away from CBD, and $n_j(r)$ the equilibrium density function of the household population in jurisdiction $j \in \{e,w\}$. Suppose in equilibrium the residents of the land at distance r in jurisdiction j are type i households. If the equilibrium level of utility of the type i agent, $i \in T$ is u^*_i, then $n_j(r) = \frac{L(r)}{s(r,u^*_i,.)}$. Let $\bar{N}_{SH}$, $\bar{N}_{SL}$,$\bar{N}_{UH}$, $\bar{N}_{UL}$ denote the total populations of the respective household types. The population constraint for each type can then be stated as:

$$\int_0^\infty \frac{L(r)}{s_w(r)} I[t^*_w(r) = i]dr + \int_0^\infty \frac{L(r)}{s_e(r)} I[t^*_e(r) = i]dr = \bar{N}_i \tag{5}$$

where $t^*_j(r)$ is a function showing the type of the occupant at distance r in jurisdiction j, and $I[.]$ is an indicator function that takes the value 1 when the condition in brackets is satisfied, and 0 otherwise. The population constraint implicitly assumes the land market clears in each jurisdiction ($\forall r \leq r^*_{fj}, s_j(r)n_j(r) = L(r)$). An equilibrium property of our model is that in each jurisdiction there is a distance r^*_{fj} called the *fringe distance* beyond which no households reside.

2.3 Neighborhoods and Schools

The focus of this paper is school quality and how various governmental policies affect this. The two jurisdictions differ only in the *quality of education* and *property tax rate* (q_j, τ_j) packages they provide.[14] There is one public school in each jurisdiction, to which only jurisdiction residents can attend. Admission is free, schools

[12] If as a result of a policy, the number of types increases, so will the number of bid-rent curves.

[13] We verify the single-crossing property of the bid rent curves numerically.

[14] de Bartolome and Ross (2003) provide an alternative model with a circular central city surrounded by a donut shaped suburban ring. With this structure, the radial symmetry permits straightforward analytical solutions of location where it is necessary only to trace locational choices along any ray from the employment center. For our purposes, however, this circular structure is problematic. For analytical purposes, all locations close to the employment center are served by a common

are financed by property taxes on residential land. In each community the entire revenue from property taxes is spent on education. Given the equilibrium rent function $R_j(r)$, and equilibrium tax rate τ_j, we can calculate the tax base and total tax revenues to find the per-student expenditure in the public school system:

$$E_j = \frac{1}{N_j} \quad \tau_j \int_0^{r^*_{fj}} R_j(r) L(r) dr \tag{6}$$

for $j \in \{e, w\}$ where N_j denotes the number of students in public school j.

The quality of education $q(\Pi, E)$ in a school (public or private) is determined by (per-student) instructional expenditures E and peer quality Π. For a given group of students, an increase in the instructional expenditures increases the quality of education ($\partial q/\partial E \geq 0$). Different groups of students may benefit differently from a given amount of instructional expenditures. That is what the peer quality (or efficiency) component captures ($\partial q/\partial \Pi \geq 0$). Some parents value education more than others, and as a result they may spend more time helping with their child's homework, provide a nicer study environment at home, be more involved in how schools operate, etc. Recall that type SH and UH agents value education more than type SL and UL agents, and as a result having more students from high valuation families may bring in a higher level of positive externality through the peer group effect. Let N_H^j and N_L^j denote the total number of high-valuation and low-valuation students in school j. The following formulation has proved to be tractable and captures the idea that the peer quality is increasing in the proportion of high valuation households:[15]

$$\Pi_j = \Pi(N_H^j, N_L^j) = c_1 \ \exp\left(-c_2 \frac{N_L^j}{N_H^j + N_L^j} + c_3 \frac{N_H^j}{N_H^j + N_L^j}\right), \qquad c_1, c_2, c_3 \geq 0. \tag{7}$$

for $j \in \{e, w, pr\}$. We adopt the specification $q_j = q(\Pi_j, E_j) = \Pi_j E_j$ for the quality function.

The private school is financed by tuition revenues, can admit students from both jurisdictions, charges the same tuition to every student, and has an objective of filling its fixed capacity $\bar{N}$.[16] If N_L^{pr} and N_H^{pr} are the number of high and low valuation

school district, making it difficult to see the separate influences of location and school quality. Additionally, there are empirical reasons to consider alternative depictions. See Hanushek and Yilmaz (2010) for a detailed discussion.

[15] Alternative specifications give similar results. See, for example, Hanushek and Yilmaz (2007) and Leung, Sarpça, and Yilmaz (2009).

[16] Private school enrollment in the U.S. has been stable around 12% in the past three decades. Empirical studies document that the reforms increasing state financing did not affect private school attendance significantly (For instance, see Sonstelie et al. (2000) for a discussion of California). Almost 80% of private school students are enrolled at institutions with a religious orientation or

households enrolled in the private school, regardless of student composition, per-student expenditure is equal to the tuition:

$$E_{pr} = \frac{pN_H^{pr} + pN_L^{pr}}{N_H^{pr} + N_L^{pr}} = p \tag{8}$$

Let $N_i^*(\Pi, p)$ denote the number of type i households that find it optimal to drop out of public schools and enroll in the private school in equilibrium, when peer quality is Π and tuition/expenditure is p.[17] Note that households would not be interested in the private school unless its quality is sufficiently higher than those of public schools so that it is worth paying the tuition. In equilibrium, the private school admits only high-valuation students because of their positive contributions to school's quality.[18] However, given any peer group, increasing quality in the private school is possible only by increasing tuition. Therefore, everything else constant, $N_i^*(.,p)$ has a positive intercept and is nondecreasing in p in the range of our interest.[19] The equilibrium tuition/expenditure level p_{pr}^* and the composition of students (N_{SH}^*, N_{UH}^*) are determined according to the following condition that ensures individual rationality and market clearance:

$$N_{SH}^*(\Pi(\bar{N},0), p_{pr}^*) + N_{UH}^*(\Pi(\bar{N},0), p_{pr}^*) = \bar{N} \tag{9}$$

If the private school admits $\bar{N}$ high-valuation students, peer quality will be $\Pi(\bar{N},0)$. Equilibrium tuition p_{pr}^* is the one that attracts $\bar{N}$ high-valuation students to the private school with this peer quality.

purpose (U.S. Department of Education 2009). This characteristic indicates the general non-profit character of the private sector and may help explain why it is not responsive to changes in school-finance policies.

[17] $N_i^*(.)$ is determined by households' solutions to (2), and thus also depends on some equilibrium outcomes that are beyond the choice of the private school (e.g., utility levels, rents, quality of public schools), which we suppress in the notation for convenience.

[18] See (7). Also, the willingness to pay for private school of SH (UH) types would always exceed that of SL (UL) types since high-valuation types have stronger preferences for school quality.

[19] With tuition/expenditure level up to \$5000 and capacity less than 20%, see footnote 16.

The most preferred tax rate for a type i household $\tau_i^* = \frac{\alpha_i}{\eta_i - \alpha_i}$ is the solution to the indirect utility maximization problem:[20]

$$\tau_i^* = \underset{\tau}{\text{argmax}}\ V(.) = \frac{k_i}{(1+\tau_j)^{\eta_i} R(r)^{\eta_i} W_i^{\delta}} q_j^{\alpha_i} (Y_i(r) - p)^{\eta_i + \gamma + \delta} \qquad \text{s.t. } q_j = \Pi_j E_j \text{ and } E_j = \tau_j \bar{R}_j \tag{10}$$

The timing of events is as follows: At the beginning of each period, households make residential choices and schooling decisions, expecting last period's quality-tax rate packages to prevail. They move in and vote for the tax rate. Households are myopic when voting; they do not consider the potential changes in the compositions of neighborhoods that can follow voting outcomes. The quality-tax rate package may be different from what they expected, but they are stuck until the beginning of next period. Then they update their expectations and the events start over again. We solve for the stationary equilibrium, which is attained when no one has an incentive to relocate in response to the voting results.

Definition: *An **equilibrium** is a set of utility levels* $\{u_{SH}^*, u_{SL}^*, u_{UH}^*, u_{UL}^*\}$*, market rent curves* $\{R_e(r), R_w(r)\}$*, quality of education and property tax rate pairs* $\{(q_e, \tau_e), (q_w, \tau_w)\}$*, quality of education and tuition for the private school* $\{q^{pr}, p\}$*, household population distribution functions* $\{n_e(r), n_w(r)\}$*, and location-type functions* $\{t_e^*(r), t_w^*(r)\}$ *that show the equilibrium occupant at distance* r *in community* $j \in \{e, w\}$ *such that:*

- *Households' choices are determined by solving* (2)*,*
- *The market rent function* $R_j(r)$ *in each jurisdiction is determined through a bidding process among different types of households,*
- *Same types of households obtain the same level of utility regardless of their choices,*
- *The private school's tuition and student composition is as described by equation* (9)*,*
- *The tax rates in each jurisdiction are determined by majority voting by myopic voters,*
- *Local governments' budgets balance in each jurisdiction,*
- *Labor and land markets clear,*
- *The population constraints* (5) *hold.*

[20] For a household choosing the public school, the indirect utility function $V(.)$ is decreasing in the property tax rate τ while increasing in q, a function of τ. When $\eta > \alpha$ the preferences of this household are single-peaked. For a household choosing the private school, q is independent of τ, so indirect utility is decreasing in τ, i.e., single-peaked with peak at zero. These ensure the existence of a voting equilibrium.

2.4 Parameters for the Computational Model

The equilibrium of our model can only be calculated numerically. We specify parameter values to match certain statistics from mid-size U.S. cities in 2005. Normalizing the sum $\alpha_i + \eta_i + \gamma + \delta$ to 1, the solution to the household problem gives the optimal budget shares for leisure, consumption, and lot size as $\frac{\delta}{\eta_i+\gamma+\delta}$, $\frac{\gamma}{\eta_i+\gamma+\delta}$, and $\frac{\eta_i}{\eta_i+\gamma+\delta}$ respectively. In the U.S., average hours of work per week in full time jobs is 40 hours, and average annual earnings of workers in 2004 are \$22,154 for high school graduates and \$38,112 for college graduates.[21] Accordingly, we set the hourly wages for unskilled and skilled households as $W_U = 15$ and $W_S = 21$. In a 168 $(= 24 * 7)$ hour week, 40 hours of work implies a 0.762 budget share for leisure. The data on household expenditures suggest that expenditures on shelter constitute about 20 percent of the budget of an average household.[22] Therefore, we set the budget share of composite commodity and land as (1-0.762)*0.8=0.1904 and (1-0.762)*0.2=0.0476. There are two possibilities for the most preferred property tax rate according to (10). We set these most preferred tax rates equal to 2.2 percent $(1.60$ percent$)$ for the high (low) valuation households. The average population density in a city with population 1 to 2.5 million is 2901 people per square mile.[23] The utility function parameters consistent with all these are $\alpha_L = 0.014$, $\alpha_H = 0.021$, $\delta = 0.75$, and $\gamma = 0.2$.[24]

We calculate the commuting costs assuming the households drive to work. The pecuniary cost can be calculated based on the cost of owning and operating an automobile. In 2004 pecuniary cost per mile was \$0.56, and we set $a = 1.1$. Assuming the commuting speed in the city is 20 miles per hour, we set $b = 0.13$. We assume 2 million households populate the city. When computing the equilibrium, we target for a (endogenous) fringe distance (city radius) of approximately 15 miles in each jurisdiction. The proportion of college graduates in U.S. is about 30 percent. We expect this proportion to be slightly higher in a city. Hence, we set the proportion of skilled households to 40 percent, about 30 percent of which are low valuation households. We assume the proportion of low valuation households among the unskilled to be higher and set that equal to 70 percent.

[21] Current Population Reports 2005, U.S. Bureau of the Census.

[22] See footnote 6.

[23] US Census Bureau, Census 2000 Summary File 1.

[24] Property taxes are paid over property value -the present value of rental income streams- whereas the model is written for a day. Therefore the property tax rates are converted to daily values, which are consistent with these parameters. See footnote 7 for details on conversion.

Private schools are assumed to be able to accommodate 12 percent of students, reflecting current urban patterns of attendance.[25] We set the parameters of the school quality function to $c_1 = 4.06, c_2 = 3.35$, and $c_3 = 0.26$ to match some related empirical observations.

3 Baseline Distribution of Households and School Quality

The previously described model is solved numerically to find the equilibrium for households and the resultant outcomes for school cost and quality. Importantly, within this model it is also possible to derive the impact of having the private school option available – something that cannot be readily analyzed using available school and locational data.

3.1 Properties of Baseline Equilibrium

The equilibrium exhibits features from both urban and Tiebout models, and the distribution of households within and across neighborhoods is more realistic than either model's implications.[26] Figure 1 and Table 1 illustrate some properties of equilibrium. Without loss of generality, we refer to the higher tax neighborhood as the *East* school district throughout the rest of the paper.

The market rents in each jurisdiction decrease in distance to the CBD, an implication of costly commuting. Significantly, in the baseline equilibrium, all four types of households are observed in both districts. Within each district, households sort with respect to distance by their type, locating in different semi-rings around CBD. The "ring" structure, dating back to von Thunen's model of land use (1826),

[25]The proportion of enrollment in private schools has increased slightly over the past decade. In 1999, 11.7 percent of total urban enrollment was in private schools; this increased to 12.6 percent in 2007 (Total private enrollment is somewhat less, given the lower concentration in rural areas). These calculations also ignore the two percent of urban enrollment that is homeschooled (U.S. Department of Education (2009)).

[26]Tiebout framework suggests that households should stratify into communities by their income and tastes, and predicts the same type households would live in the same community. Davidoff (2005), among others, reports that communities are empirically heterogenous. Urban models predict households sorting with respect to distance around the CBD. Glaeser, Kahn, and Rappaport (2008) suggest that observed residential behavior exhibits more stratification than these models can support. Nechyba (2000,2003) facilitates community heterogeneity similar to ours without modeling space, only by imposing a fixed heterogenous housing stock in each neighborhood, thus restricting neighborhood compositions and tax bases to a great extent. In Epple and Platt (1998) communities consist of multiple types of households, but a certain type will only be observed in a single community.

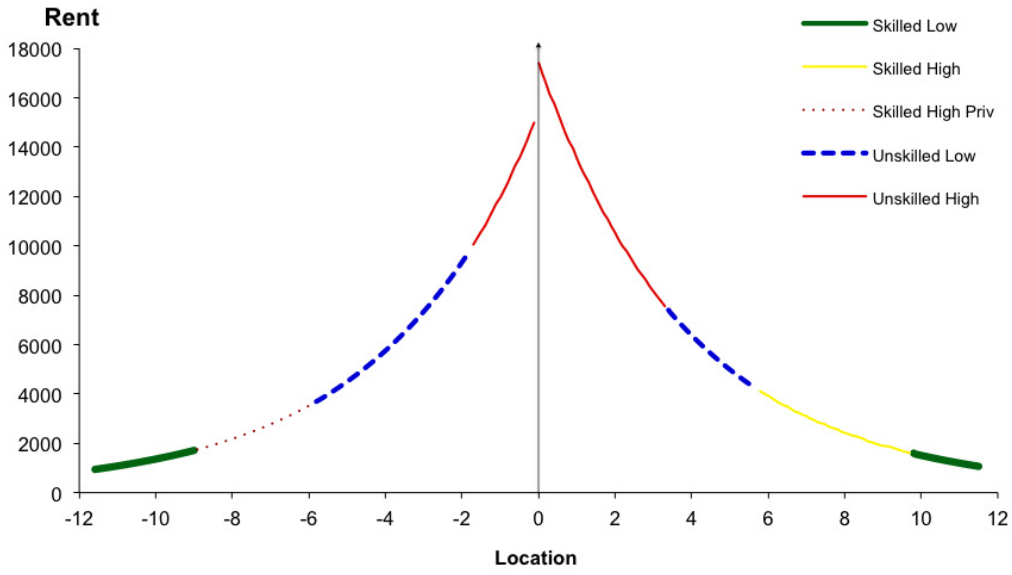

Figure 1: Rents and Spatial Distribution of Households in the Baseline Equilibrium

is replaced by semi-rings here because of the jurisdiction boundaries that identify differing bundles of the local public good. In general, the widths and numbers of rings will show variation in the two localities.[27] The ordering of households around CBD, however, is same in each district: Among households with same income, a stronger preference for education (a weaker preference for lot size) causes households to locate on smaller lots closer to CBD. Similarly, among households with same preferences, higher income increases the land demand and leads households to reside further away from CBD where the rents are lower. High income also increases the opportunity cost of commuting time, but our calibration suggests that the land demand effect dominates, consistent with residential patterns observed in the U.S.[28] The resulting ordering of households (starting from CBD) in each district is as follows: UH, UL, SH, and SL. In general, if a particular household type is not present in a district because of the pattern of public good provision, the order between the others would still be preserved.

[27]Higher school quality in one neighborhood may substitute for accessibility in the other neighborhood, so households may be indifferent between living at distance r in a certain neighborhood and $r+d$ in a higher quality neighborhood (holding lot size constant).

[28]Glaeser, Kahn, and Rappaport (2008) show that including low-cost public transportation options around the city center would result in similar residential patterns, which can be supported by a lower income elasticity of demand for space. Since our focus in this paper is on educational outcomes resulting from the residential patterns, we abstract from the complexity of distance-varying transportation choices.

Table 1: Equilibrium Location, Spending, and School Quality for the Baseline Equilibrium

	West	East	Private
School Quality	10.9	14.2	23.4
Expenditure per pupil	$ 2701	$ 2414	$ 2040
Property tax rate	1.60%	2.20%	
Gross Rent (per acre)	$ 2930	$ 3332	
Distribution of households across schools/districts			
SH	-	57.1%	42.9%
SL	58.6%	41.4%	-
UH	23.9%	76.1%	-
UL	58.8%	41.2%	-
Distribution of households within schools/districts			
SH	-	30.4%	100%
SL	19.9%	9.4%	-
UH	13.6%	29.0%	-
UL	66.5%	31.2%	-

Note: SH: Skilled household with high education demand; SL: Skilled household with low education demand; UH: Unskilled household with high education demand; UL: Unskilled household with low education demand.

We also observe a partial sorting according to preferences across districts. More of both SH and UH households reside in the east rather than the west, and high valuation households constitute the majority there. As a result, the property taxes and quality of education are higher. School quality is capitalized into rents, therefore rents in the east are higher too.

The equilibrium tuition/expenditure level at the private school is also intuitive. Private school choice is such that with a lower level of tuition/expenditures the private school could not offer a quality that makes it optimal for enough households ($\bar{N}$, see (9)) to drop out of free public schools and pay its tuition. This implies that the equilibrium tuition level will respond to policies that alter the quality of public schools or households' ability to pay, as we discuss below.

In general, the most preferred tax rate for a household with a child in the private school is zero, therefore all households that choose the private school choose to reside in the lowest tax neighborhood. In the equilibrium of this baseline model, the private school consists of just students from SH households, and these SH households in private school are the only SH households that locate in the west school

district.[29] Note that the per-student expenditure in the west school district is higher than that in the east. The SH residents who attend the private school contribute significantly to the district's budget but do not claim any share of the revenues. This increases the per-student expenditure for those at the public school. As a result, the west district actually spends more per pupil, even though it has a lower tax rate. However, higher expenditures fail to provide a higher quality than in the east. The private school attending students are desirable (high valuation) peers, and with them being out of the peer group, the public school consists mostly of low valuation households. Because of peer quality, the private school obtains a higher quality than both public schools even though it has a lower expenditure level.

Most low income-high valuation (UH) type households choose to live in the east –despite the higher rents– because of the higher quality of public education. This highlights the fiscal burden problem present in all models we study in this paper: On average, the per-student expenditures in a district are lower than what SH households pay in taxes, while the opposite is true for the UH households.[30] This redistribution illustrates why the high income residents often try to keep low income residents out through such devices as minimum lot size zoning.[31] For low income-low valuation type households, the lower rents and taxes in the west compensate for the lower quality of education.

3.2 Equilibrium without the Private School

To gain further insight into the equilibrium effects of private schools, we consider the public schools only equilibrium, i.e., the equilibrium when private schools are prohibited from entering. Table 2 displays some basic equilibrium properties. In the new equilibrium, the differences in school quality, rents, educational expenditures, and peer quality across the two districts are largest among all models we study in this paper. The quality of education in the lower tax neighborhood is disturbingly low.

[29]Note that our equilibrium has high income private school households living in the low rent jurisdiction that houses a disproportionate share of the poor households. For the U.S. as a whole, the correlation of income and private school share by school district was 0.28 in 2000. This correlation implies a large variation in the pattern of residences by private school attendance, but it also suggests that parents are looking for neighborhood amenities in terms of higher income/higher valuation households living nearby.

[30]Note that the issue of fiscal burden cannot readily be analyzed in the standard Tiebout or residential location model because homogeneity of communities rules this out.

[31]For a discussion of zoning issues, see Calabrese, Epple, and Romano (2007); Fischel (1985); and Hamilton (1975).

Table 2: Equilibrium Location, Spending, and School Quality without Private Schools

	West	East
School Quality	6.4	18.7
Expenditure per pupil	$ 2012	$ 2427
Property tax rate	1.60%	2.20%
Gross Rent (per acre)	$ 2698	$ 3663
Distribution of households across schools/districts		
SH	-	100%
SL	100%	-
UH	-	100%
UL	63.4%	36.6%
Distribution of households within schools/districts		
SH	-	44.6%
SL	32.1%	-
UH	-	32%
UL	67.9%	23.4%

Note: SH: Skilled household with high education demand; SL: Skilled household with low education demand; UH: Unskilled household with high education demand; UL: Unskilled household with low education demand.

When schooling alternatives consist of the two public schools only, households respond by sorting more completely according to their preferences. An inspection of Table 2 makes this clear. All SH households and all UH households locate in the east, i.e., some semi-rings disappear. All SL households and most UL households locate in the west. This causes a large difference in peer quality across districts. Also, without the private school attending SH households, the per-student expenditure in the west is much lower, reflecting the fact that all tax payers now consume public schooling.

The per-student expenditure in the east is only slightly higher than in the benchmark. With all UH households locating in the east, the fiscal burden on SH households is larger. Even though peer quality (hence the effectiveness of each dollar in the school budget) increases, this incentive is insufficient to lure significant enough increases in the SH households' willingness to pay for land, and this in turn limits per-pupil expenditure.

School quality in the east does benefit the high valuation households, all of whom reside there now. However, the increases in rents and tax expenditures outweigh this quality increase, and SH households end up worse off. In the benchmark, the fiscal contribution of private school attending SH residents was a major cause

for UH households choosing to live in the west. Now, the UH households move to the east for a better education. However, the increase in quality of education fails to compensate for the higher rents and taxes for them also. Similarly, the rents decrease in the west school district. The residents are (skilled and unskilled) low valuation households exclusively. However, the decline in the quality of education is so dramatic, it outweighs the gain from rents and taxes. In equilibrium every household is worse off compared to benchmark model.[32]

* * *

The private school weakens the Tiebout sorting and leads some high income households with strong preferences for education to choose residences in the area with relatively lower quality public schools but also lower taxes. These parents increase the per-student expenditure in the district by paying taxes higher than the typical resident and also by not claiming their share of local expenditures, since their children attend the private school. As a result, the other residents of poorer districts can have a higher quality in public schools than they could in the lack of the private school.

4 School Policy Alternatives

The use of local property taxes to support schools is largely a unique American institution, and one that opens up the possibility of clear disparities in school quality. The more common alternative around the world is full funding from the centralized government. In our simplified model, we consider this case, where equal funding to the districts is provided (through a uniform property tax). An alternative policy approach is to reinforce and increase the ability of households to choose their schools through the provision of vouchers. Our analysis considers both a general voucher and an income-targeted voucher plan.

We begin with a discussion of these options, and then we present a comparison of the welfare implications across the policies next. Additionally, because the discussion of these policy options invariably focuses explicitly on the educational outcomes, we also summarize the level and distribution of education that results from each policy.

[32] Quantitative welfare comparisons of all models in this paper are presented below in Section 4.3.

4.1 Full State Funding

Disparities in local schools – both of funding and of outcomes – have led to pressures to move away from such reliance on the local property tax and on local funding. One obvious alternative is to have full state funding, where funds are raised centrally and distributed to local jurisdictions. Another version that is very closely related would be district consolidation, where the two school districts are united to form a single large school district.[33] These two policies could be similar or distinctly different, depending on the rules of school attendance. Here we consider the simple case where there are specific and binding school attendance boundaries (following our previous jurisdictions). Also, since we consider a single metropolitan area, there is no distinction between state and area policies, so we place this within the context of district consolidation.

The two public schools now have a common budget, and the majority voting is conducted area-wide. However, the school attendance boundaries still allow for peer quality differences between the two public schools. Comparing the equilibrium of this model to that in the benchmark allows us to study the general equilibrium effects of school district consolidation in the presence of a private sector for education.

Table 3 displays some equilibrium properties. Both the quality of education and the per-student expenditures at public schools are lower than those of the benchmark equilibrium. The quality gap between the two public schools is slightly larger compared to benchmark.

The low valuation households constitute the majority in the population, implying that the area-wide fiscal voting outcome is the lower tax rate. Combined with the decrease in rents in both communities, this results in lower tax revenues overall and explains the low expenditures. In the benchmark model, households recognize that the quality of education in the local public school is affected by instructional expenditures and peer effects, and they make locational and tax decisions accordingly. But now they are left without tax and spending choices, which cannot be as effective as also allowing this margin of choice. Further, in the benchmark model, the tax revenues from private school attendees that live in the west contributed to the budget of public school students in the west only. Many unskilled high valuation households chose to live there because of the resulting high per-student-expenditure. Now the SH households that attend the private school still live in the west because of the lower rents, however their taxes no longer benefit west community exclusively,

[33] School district consolidation policies are generally motivated by economies of scale, aiming to increase efficiency of public school systems by lowering administrative costs. Over the last several decades, the number of school districts has decreased dramatically in the U.S. While there once were over 100,000 districts, this has dropped to less than 14,000 in 2008 (U.S. Department of Education 2010).

Table 3: Equilibrium Outcomes with Full State Funding of Public Schools

	West	East	Private
School Quality	9.2	13.4	16.0
Expenditure per pupil	$ 2278	$ 2278	$ 1395
Property tax rate	1.6 %	1.6 %	
Gross Rent	$ 2875	$ 3111	
Distribution of households across schools/districts			
SH	-	57.1%	42.9%
SL	47.4%	52.6%	-
UH	13.7%	86.3%	-
UL	61.9%	38.1%	-
Distribution of households within schools/districts			
SH	-	29.2%	100%
SL	17.1%	11.5%	-
UH	8.3%	27.8%	-
UL	74.6%	27.8%	-

Note: SH: Skilled household with high education demand; SL: Skilled household with low education demand; UH: Unskilled household with high education demand; UL: Unskilled household with low education demand.

since east and west have a common budget. There is no difference in expenditures between two districts, but the peer quality in the east is higher. This affects the incentives of UH type residents, and more of them reside in the east.

This low quality of public education in both localities makes the private school much more attractive to all high valuation households. At every tuition/expenditure level, more households are interested in the private school compared to benchmark (i.e., N^*_{SH} and N^*_{UH} shift, see (9)), and this shift in demand fills the private school at a lower tuition/expenditure/quality level than that in benchmark. We see the most dramatic effect of this policy on the quality of private school.

The combined effects not only decrease the rents in both jurisdictions, but also narrow the rent gap between the communities. East housing is relatively cheaper compared to the benchmark, and this attracts some new residents - particularly SL households. Despite the decrease in quality of education, every household will benefit from the decrease in rents and tax payments. To see which effect dominates, a welfare analysis indicates that the impact of the consolidation (or full state funding) is not only negative for everyone, but also huge in magnitude. Table 6 below compares the welfare changes across policies and finds the losses far exceed those for prohibiting private schools or for the alternative voucher policies discussed below. In our economy low valuation households are the majority so the tax outcome rep-

resents their preference. This hurts high valuation households and triggers a chain of events –some of which we described above– which in turn hurts the remaining households in equilibrium.

4.2 Extending the Ability to Choose

Our analysis so far suggests that restricting choice may actually worsen the distribution of outcomes. The quality difference between the east and west public schools increases when they are merged into a single district (or funding is restricted to be the same), and the quality difference increases dramatically when there is no private school. These observations motivate us to consider policies that seek equity through extending low-income households' ability to choose, instead of restricting alternatives for higher-income households. In practice, this would be desirable not only in terms of fairness and equality, but also in terms of increasing school competition and efficiency. If choice is beneficial, then its effects could be amplified as the number of people that can exercise choice increases. In this section, we investigate the equilibrium implications of two programs that aim to enable more people to exercise choices through forms of vouchers or charter schools.[34]

4.2.1 Property-Tax Financing

Under the common institutional structure, public schools are financed by property tax revenues, and those that choose the private school give up any claim on these revenues, effectively subsidizing others (e.g., SH households in the west at benchmark). As an alternative policy, the districts may use property tax revenues to provide support also to their residents that choose the private schools. This policy may enable more families to compete for private schools, while allowing private school attending households to benefit from the taxes they pay. This policy actually is analogous

[34]Within the range of voucher programs, school choice advocates have proposed a variety of alternatives, and we analyze a subset of these. Vouchers can be unconditional, or eligibility and voucher amount can be conditioned on some student characteristics (income, ability) or school characteristics (tuition). Different types of vouchers would result in different distributions of students across schools. Voucher design is a very interesting problem beyond the scope of this paper (see Epple and Romano (2008), Necyba (2000), Caucutt (2002)). The mechanism we consider in 4.2.1 can be thought as a flat-rate voucher, to which every household is eligible. The mechanism in 4.2.2 can be thought of as a targeted voucher. For prior discussions of the effects of vouchers, see Manski (1992), Epple and Romano (1998), Rangazas (1995), Hoyt and Lee (1998), Nechyba (1999, 2000) among others.

to a district-supported charter school, where revenues go directly to supporting an alternative choice school.[35] The policy differs from normal charter schools, however, because tuition adjusts to achieve a fixed number of students in the alternative (private) school; in charter schools, if an excess demand for attendance is seen, spending remains constant but lotteries are used to choose who can attend and the school does not have the ability to choose on characteristics of the student.

Formally, the district offers a transfer V_j that can be used for tuition p, to households whose children are admitted to the private school. This transfer amount cannot exceed the district's per-pupil expenditure $\widehat{E}_j^{pub}$ in the public school system:

$$V_j = min\{p, \widehat{E}_j^{pub}\}, \qquad j \in \{e, w\}. \tag{11}$$

Then a household with a student in the private school has the budget constraint:

$$z + (1+\tau)R(r)s + Wl + max\{p - \widehat{E}^{pub}, 0\} = 24W - (a + bW)r. \tag{12}$$

Let $\widehat{N}_j^{pub}$ and $\widehat{N}_j^{pr}$ denote the numbers of district j residents with children attending the public and private schools under this policy. The per-student expenditure in public school of district j is then the $\widehat{E}_j^{pub}$ that solves:

$$\widehat{E}_j^{pub} = \frac{1}{\widehat{N}_j^{pub}} \quad \left[\tau_j \int_0^{r_{fj}^*} R_j(r)L(r)dr - \widehat{N}_j^{pr} min\{p, \widehat{E}_j^{pub}\}\right] \tag{13}$$

The equilibrium implications of this policy can be seen in Table 4. The quality difference between the two public schools increases. The school in the east (west) has higher (lower) quality compared to the benchmark equilibrium. The expenditures in the two districts differ by a small amount; the quality difference is caused mainly because of the stronger sorting according to preferences affects the peer composition of the schools.

This program increases every household's ability to pay for the private school. In fact, if the tuition/expenditure level in equilibrium is lower than $\widehat{E}_j^{pub}$, the private school is free to admitted households. Our analysis reveals that demand by UH households for the private school equals supply at the modest level of tuition/expenditure of \$859.[36] Even though a tuition increase up to $\widehat{E}_j^{pub}$ comes at no cost to admitted

[35] Vouchers are frequently proposed to promote efficiency through forcing schools to compete for students. We do not consider such an effect here. In fact, choice operates quite differently in our model because increased efficiency in one school (because of peers) implies a decrease in efficiency at the other school.

[36] To attract SH types, tuition/expenditure level (hence the quality) needs to increase at the private school. But any tuition/expenditure level lower than $\widehat{E}_j^{pub}$ will be financed by the program, and the

Table 4: Equilibrium Outcomes with Local Property Tax Funding of School Vouchers

	West	East	Private
School Quality	8.5	16.1	9.9
Expenditure per pupil	$ 2314	$ 2513	$ 859
Property tax rate	1.6 %	2.2 %	
Gross Rent	$ 1582	$ 3079	
Distribution of households across schools/districts			
SH	-	100%	-
SL	89.7%	10.3%	-
UH	-	39.5%	60.5%
UL	55.2%	44.8%	-
Distribution of households within schools/districts			
SH	-	50.9%	-
SL	32.8%	2.3%	-
UH	-	14.2%	100%
UL	67.2%	32.6%	-

Note: SH: Skilled household with high education demand; SL: Skilled household with low education demand; UH: Unskilled household with high education demand; UL: Unskilled household with low education demand.

students, it would result in excess demand by them, violating the equilibrium condition (9). The resulting quality is very low, closer to that of the public school in the west. The peer quality did not diminish, since all students are still high valuation households, so the quality decline is caused by the big reduction in per-student expenditure.

When UH households replace the SH households in the private school, the west loses its SH residents and their contributions that generate the high expenditures in the benchmark equilibrium. Then the UH households that do not attend the private school move to the east too, and the west public schools consist exclusively of low valuation students. On the other hand, the number of UH households in the east decreases significantly when compared to benchmark, since a majority of them attend the private school and choose to live in the west. This decreases the fiscal burden on SH households in the east, increasing their willingness to pay for land

competition (and excess demand) from UH will prevail. So SH types can take over the private school only at very high tuition/expenditure levels. Note that this would imply more UH types residing in east (as in benchmark model), increasing the fiscal burden on SH types residing there. For all these reasons, SH types are content with the current allocation.

there. As a result, the gaps in spending and peer quality between the two districts increase compared to benchmark.

The rents in the west decrease substantially. However, our welfare analysis indicates that this decrease in rents does not make up for the decrease in quality of education. The unskilled high valuation households do benefit from this policy, but the other three types of households lose (see Table 6 below). This is not surprising since more than 60 percent of the UH attend the private school without paying tuition while residing in the low-tax/low-rent neighborhood; the remaining 40 percent live in the east with a higher quality of education and less competition by UH households for land. The shifts in rents, taxes, and school quality, however, leave the other three types of households worse off compared to the benchmark model.

4.2.2 Targeted Vouchers

Note that the local-financing structure in the prior case makes poorer west residents bear the cost of this program, since households attending the private school always choose to live in the neighborhood with lower property taxes. This observation motivates us next to study the implications of an income-tax financed alternative that effectively means there is provision of a state-funded voucher. Here we also move explicitly to an income-targeted voucher.

Consider a uniform income-tax that finances a voucher to all the unskilled residents in any locality, conditional on admission to the private school. We denote the (exogenous) income-tax rate by θ. The budget of a typical household is then:

$$z+(1+\tau)R(r)s+p=(24-l)(1-\theta)W-(a+b(1-\theta)W)r.$$

In addition to all the standard effects of an income-tax, note the decrease in commuting costs caused by the decrease in the opportunity cost of time. The total income-tax revenue is distributed equally among all *unskilled* households that are admitted to the private school.

This program increases the ability to pay for the private school by unskilled households, facilitating their competition with skilled households. The choice of the income-tax rate gives the policymaker a control over the extent of this competition, and the resulting student composition at the private school. Here we report results from one of the more interesting "mixing" cases: When $\theta = 0.26$ percent, both UH and SH households can be observed in the private school. As we decrease the tax rate, the proportion of UH households in the private school decreases, and the equilibrium converges to that of the benchmark model. If we increase the tax rate, the proportion of UH households in the private school increases, resulting in an equilibrium similar to the one under property-tax financing.

Some key properties of the equilibrium are presented in Table 5. The school in the east (west) has higher (lower) quality compared to the benchmark equilibrium, and the quality of private school is lower than that in benchmark, similar to the property-tax financed program discussed above. But the quality of education at every school is higher than that under property-tax financing.

Table 5: Equilibrium Outcomes with Income Tax Funding of Targeted School Vouchers

	West	East	Private
School Quality	9.7	16.8	17.0
Expenditure per pupil	$ 2647	$ 2586	$ 1483
Property tax rate	1.6 %	2.2 %	
Gross Rent	$ 1804	$ 3174	
Distribution of households across schools/districts			
SH	-	84.3%	15.7%
SL	70.6%	29.4%	-
UH	-	61.6%	38.4%
UL	61.8%	38.2%	-
Distribution of households within schools/districts			
SH	-	43.2%	36.5%
SL	25.5%	6.5%	-
UH	-	22.5%	63.5%
UL	74.5%	27.9%	-

Note: SH: Skilled household with high education demand; SL: Skilled household with low education demand; UH: Unskilled household with high education demand; UL: Unskilled household with low education demand.

High-valuation households –skilled or unskilled– either live in the east or attend the private school and live in the west. The public school in the west consists of low-valuation students exclusively. West rents are significantly lower compared to the benchmark, whereas rents in the east decrease only slightly. The SH households that are replaced by UH households in the private school move to the east, but this does not hurt the west expenditures much: First, about 16 percent of all SH households (about one third of the benchmark level) still choose the private school and reside in the west. Second, the low rents attract new SL residents, while the lower quality of education drive UH households to the east. As a result, except for the private school attendees, the west consists of low-valuation households with higher average income compared to the benchmark equilibrium. These low-valuation households have stronger tastes for land, and the low east rents increase

a typical household's lot size significantly, causing the neighborhood and the taxable land to expand. This also decreases the population density and the number of students in the public school, increasing per-student expenditures. These limit the decrease in school quality in the west.

The UH households who do not attend the private school live in the east, but their numbers are lower than those in the equilibrium of the benchmark model. This decreases the burden of UH households on SH households, increasing the effectiveness of SH money and SH households' willingness to pay for land in the east. The resulting expenditure level in the east exceeds the benchmark levels, and with a higher peer quality, the schooling outcomes for east residents improve over benchmark levels. All high valuation households have higher utility levels compared to benchmark equilibrium under this policy. Moreover, this program is a Pareto improvement over the program with local financing. In fact, among four models we study above, this policy is the only one that results in an improvement over the benchmark equilibrium in terms of aggregate utility (see Table 6 below). Note, however, that this is not a general result, because it depends on setting the voucher at a level that yields a private school with a mix of students. Other taxes that yield segregation in the private school by low skilled households would look closer to the prior voucher option, while a voucher too small to attract low-skill households would be closer to the baseline.

4.3 Equilibrium Welfare and School Quality

We can now summarize the overall impacts of all the models and policy simulations we consider in this paper. The unit of measurement in Table 6 is the percentage change in rent required to keep the relevant type of households at the equilibrium utility levels in the benchmark model. A negative number means the household type is worse off. The bottom row of the table displays our measure of the change in overall welfare, the change in rents necessary to keep the aggregate utility at its benchmark level. A quick inspection of the table reveals that the programs that restrict choice by eliminating private schools or removing local choice over revenue hurt all households (columns 1 and 2). This suggests that the benefits of choice and autonomy extend even to households that choose the public schools in lower spending neighborhoods. Also, programs that expand choice (columns 3 and 4) should be financed by the state rather than the districts.

Table 6: Welfare under Alternative Policies Compared to Benchmark

Type	No Private School	Full State Funding	District Vouchers	State Vouchers
SH	-0.09	-3.18	-2.05	0.14
SL	-0.02	-2.65	-1.03	-0.21
UH	-0.09	-3.60	1.04	1.96
UL	-0.16	-2.81	-1.65	-0.07
Aggregate Utility	-0.11	-2.97	-1.15	0.35

Note: SH: Skilled household with high education demand; SL: Skilled household with low education demand; UH: Unskilled household with high education demand; UL: Unskilled household with low education demand.

When there are no private school alternatives, households respond by a stronger sorting according to their tastes for education. This results in the isolation of the low-income and low-valuation households in the east, and a concentration of unskilled high-valuation households in the west, which in turn increases the fiscal burden on skilled high-valuation households since they carry a greater school financing load. The quality of education increases for high-valuation households as a result of stronger sorting, but they also pay higher rents as a result of stronger competition for east land. We find that the utility losses from higher rents outweigh the utility gains from the increase in education quality.

Full state funding decreases the quality gap between the two neighborhood schools as expected. However, this results in lower quality levels at all schools and substantial decreases in equilibrium utility levels for every household type. In general, Tiebout sorting results in an efficient provision of local public goods. In communities consisting of households that have similar preferences and willingness to pay for the public goods, the provision level represents the community members' most desired levels, and the property tax becomes a fee, eliminating deadweight loss. Any restriction on Tiebout sorting would have an efficiency cost and would also increase the demand for the private school at every tuition/expenditure level.

The programs that support private school tuition enable low-income high-valuation households access to the private school. Other households respond with a stronger sorting across neighborhoods according to their tastes for land and for education. A noteworthy feature of these programs is that they decrease fiscal burden of the unskilled (UH) households on skilled high-valuation (SH) households, increasing the latter's willingness to sort and their willingness to pay for east land. The resulting increase in the school quality difference between the two districts also increases the rent gap between the districts. The utility gains from the decrease in west rents compensates partly for the utility loss to low-valuation households arising

from education quality. Both property-tax financing and income-tax financing cause some losses in low-valuation households' welfare, but property-tax financing also places the financial burden of this mostly on low-valuation households. Income-tax financing results in an improvement over the benchmark equilibrium in terms of aggregate utility.

Table 7: Average Educational Outcomes by Income and Taste Groups

	Benchmark	No Private School	Full State Funding	District Vouchers	State Vouchers
Skilled	16.4	15.0	13.6	14.1	15.3
Unskilled	12.6	13.5	11.5	12.1	13.9
High-valuation	16.2	18.7	13.8	14.5	16.9
Low-valuation	12.3	9.8	11.3	11.3	12.2
All Households	14.14	14.08	12.32	12.87	14.45

It is also useful to look explicitly at the quality of education under each alternative, since these policy choices are frequently motivated by arguments about the level and distribution of school outcomes. Table 7 summarizes the average schooling outcomes by income and taste groups under the different models we study in this paper. Full state funding and district-financed vouchers lead to a deterioration in the average education outcomes for every group. The lack of a private alternative and income-tax financed vouchers bring an improvement in average education outcomes for unskilled and high-valuation groups. In the first case, when there is no private school, this improvement comes at a welfare cost to all households as discussed above. With state financed vouchers, only a subset of the households bear welfare costs, and the overall welfare increases.

5 Concluding Remarks

This theoretical investigation begins with a simple calibrated household location model where residential locations have two attributes – distance to work and tax-school bundles. To this, we add a private schooling option that, while costly, is not dependent on residential location. Our general equilibrium model starts with a two-community monocentric city, and illustrates how private schooling alters the residential and educational opportunity sets and choices of different types of households.

The role of private school choice in improving the welfare of families is under-appreciated. Much of the discussion of private schools views them only as affecting the welfare of households that choose private schools, but such a discussion

fails to understand how the effect of private schools is felt throughout the educational system. Private schools have impacts on educational outcomes through three mechanisms. First, the private schools themselves may offer a superior education for those attending them. Second, private schools may affect the peer composition in both the public and private schools, and this may affect student achievement. Third, private schools may offer competition for students that induces improved performance from the public sector – seen in terms of better meeting the demands of parents or producing education more efficiently. We focus on the first two of these, but we also consider a generally neglected part of the story, namely the fiscal implications of private school enrollment.

The fundamental conclusions are quite clear: Having a larger opportunity set has benefits not only for the households that select those alternatives, but for others as well.[37] Indeed, eliminating the private school option results in a disturbingly low quality of education in the poorer neighborhood, and full state funding brings fairly large welfare losses to every resident.

Much of the public discussion of private schools has been concerned with the inequality of opportunities between the rich and poor when choosing schools. To consider this, we have introduced two very simple voucher mechanisms to extend the ability for the latter. Our results point to the desirability of central financing of choice as opposed to local financing.

Our model also allows us to understand the fiscal outcomes better. Because rich and poor live in the same jurisdictions (unlike the outcomes in most pure Tiebout or pure residential location models), the poor can introduce a fiscal burden on the rich by paying less taxes than their proportionate share of school expenditures. This fiscal burden varies with the overall pattern of living induced by differing institutional structures. But, another fiscal aspect of private schools is the fiscal bonus that those attending private schools confer on other residents of their community, because privately educated children require no public school expenditure. Removal of this bonus with the elimination of private schools is shown to exacerbate schooling inequities that arise from normal household location choices.

From a theoretical perspective, this work also demonstrates the importance of considering multiple attributes of location. A crucial element of the modeling here is inclusion of both accessibility and public amenities (schools). With this, the baseline communities exhibit heterogeneous populations. But, importantly, the outcomes that result from the availability of a private schooling option that does not depend on location become clear. This schooling option, coupled with the maximiz-

[37]Nechyba (2000) constructs a model that is quite different than ours, one with homeowners and that abstracts from distance and costly commuting, yet predicts inter-district sorting incentives and welfare results that overlap with the findings of this paper.

ing behavior of households, reduces the segregation that occurs when schooling is directly linked to location.

We have focused on how the structure of schooling options affect the general equilibrium outcomes. By solving this model for a consistent set of parameter values, we can provide direct comparisons of the welfare and schooling outcomes that result from household choices. There are a variety of modifications and extensions that point to directions for further research. First, experimentation with alternative parameters – particularly for the household utility function and the educational production function – would provide information about the generalizability of our conclusions about choice mechanisms. Second, while we maintained a fixed supply of private school spaces (with a varying expenditure level), it would be useful to allow private schools to adjust to demand.[38] Third, a different dimension of the private school question is the implication for efficiency. In our analysis, the private school competition for students affects the outcomes of the public schools through its impact on the peer group remaining in the public schools, but the effect of this competition is zero-sum – what one sector gains, the other loses. Given that many arguments for increased competition among schools focus on the possibility of increased efficiency in operation of the public schools, it would be useful to consider how this can be incorporated.

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[38] Other analyses (e.g., Epple and Romano 1998) have gone in a different direction from our work. They have considered a single public school with competition among a set of private schools. It is difficult to analyze multiple public and multiple private schools simultaneously, but it is possible within our framework to consider the impact of a larger or smaller private school.

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