The complementarity of Tiebout and Alonso

Eric Hanushek a,*, Kuzey Yilmaz b

a Hoover Institution, Stanford University, Stanford, CA 94305-6010, USA
b Department of Economics, Koc University, Sariyer, Istanbul 80910, Turkey

Received 8 August 2006
Available online 12 June 2007

Abstract

While residential location is an important aspect of both models of urban spatial structure and local public goods, previous modeling efforts have most commonly separated these. The resulting models yield unrealistic locational predictions. This paper incorporates both motivations simultaneously and finds that the equilibrium outcomes are more consistent with empirical observation. Having a more realistic model permits analysis of current school finance proposals. Because school finance is focused on local jurisdictions, a more realistic general equilibrium model is essential to assess the impact of governmental involvement on the K-12 school system.

© 2007 Elsevier Inc. All rights reserved.

JEL Classification: H4; H7; I2

Keywords: Tiebout model; Urban location model; School finance

1. Introduction

Models of urban spatial structure have followed two distinct lines. Residential location analyses have emphasized the trade-off between accessibility and space, while public finance analyses concentrate on the differential provision of public goods and services. Unfortunately, in their attempts to simplify the structure and to focus attention on one element of family behavior, they each produce in an unrealistic description...
of key elements of urban economies. As a result, they offer questionable predictions about equilibrium outcomes, particularly when addressing a variety of policy options. By combining the essential features of the two models, we show how a unified model yields much more satisfactory descriptions of equilibrium in spatial economies. Even though the models are highly stylized, they provide suggestive results about the impacts of current school finance policy and show the importance of varying locational incentives.

The pioneer of modern urban location theory was Alonso (1964) with his model of the land market that generalized the much earlier work of Von Thünen. This work was followed with theoretical and empirical work by such scholars as Muth (1969) and Mills (1972) and has been extended in a variety of dimensions (e.g., see Straszheim, 1987; Brueckner, 1987; Fujita, 1989; Cheshire and Mills, 1999; Glaeser and Kahn, 2004; Henderson and Thisse, 2004). These models investigate the character of equilibrium and optimal land use in the context of multiple household types. In the simplest such as we employ here, employment is centrally located. Assuming that a set of bid-rent functions can be ordered according to their steepness, they imply that the equilibrium land use and optimal land use exist uniquely with households stratified by commuting distance from the Central Business District. These stylized models offer insights into urban spatial structure, and have been extended to consider multiple workplace locations, housing markets, and the like. But they also suggest a degree of residential stratification by income than is not found within urban areas, which raises questions about whether they can support analyses of jurisdictional policies.

In another stream of the literature, referred to as Tiebout models of community choice, households care about local public goods and vote with their feet to shop for the community which best satisfies their preferences. This literature has evolved from the central insight of Tiebout (1956) and builds upon the analytical framework developed in Ellickson (1973). The most influential studies from this approach have been conducted by Epple et al. (1984, 1993) and Epple and Romano (1998, 2003), who have also introduced politics into the model. Related to this research, Fernandez and Rogerson (1996) develop a multi-community model and analyze policies that affect spending on public education and its distribution across communities. This literature concludes that households should stratify into communities by their income and tastes, and predicts the same type households would live in the same community. This is an important shortcoming of these models, given that communities are empirically heterogeneous.\textsuperscript{1} The reason for this counterfactual result in Tiebout Models is that these models are essentially designed to deal with spaceless economies, ignoring spatial problems such as land use, geographical allocation of households, etc.

Some prior work has addressed the problem of homogeneous communities in Tiebout models. Epple and Platt (1998) introduce households that differ both by income and by tastes and show that there is income heterogeneity within communities because of these preference differences. In their model, they concentrate on residential location decisions where different communities provide differing amounts of local redistribution

\textsuperscript{1} In his empirical work, Davidoff (2005) reports that stratification by income, generated by the differences in tax and spending policies, into communities is far from complete. These differences account for only approximately 2\% of the variation in household income.
of income. Their results lead to an interpretation of the resulting communities as a central city (with redistribution) and suburban locations (without redistribution), but location (or accessibility) per se is not important.\footnote{With a single community characteristic (the amount of local redistribution), the distribution of tastes yields an equilibrium with communities that have a mixture of income. The same type of individual (denominated by income and taste) will only be found in a single community. As described below, when there are multiple motivations for living in a community, the same type of household can be found in different communities in equilibrium.} Nechyba (2000, 2003) begins with exogenous housing heterogeneity, which leads to community income heterogeneity, and considers how different policies affect this heterogeneity. A complete review of alternative modeling approaches is provided by Epple and Nechyba (2004) and Nechyba (2006).

This paper is partially motivated by recent policy discussions about school finance. State courts and legislatures have frequently changed the funding of local schools over the past several decades without a good understanding of how the outcomes are affected by the locational choices of households. In order to address these issues, it is necessary to have some understanding of the interaction of household location and the financing and quality of schools. While neither of the classic models of urban structure can address these issues, a combination of the two provides the basic building blocks for evaluating a variety of policies.

Our model incorporates both locational motivations—accessibility and public goods—simultaneously and finds that the equilibrium outcomes are more consistent with empirical observation. We stay with a monocentric city model of all central employment, but expand the model to contain two school districts. Households differ in income and tastes, which in turn affects how they value the accessibility, lot size, and public amenities (education) of a location. Even though school districts have the same production technology, they end up with different efficiency in how they convert spending into outcomes (or quality) due to peer group effects.

This very simple model yields equilibria that differ sharply from those found in either standard urban location models or Tiebout choice models. The two districts in fact have a mixture of incomes and people with different preferences with respect to schools. Given this baseline, we illustrate the interactions of forces by evaluation one of the most popular school reform policies of the last century—the consolidation of school districts. This example is just one possible instance of the impact of governmental reform attempts on the K-12 education system, where policy must recognize the behavioral reactions of individuals to the policy.

This analysis concentrates on the long run equilibrium for the residential location of households. As such, it ignores any of the short run dynamics or of the interactions with the macroeconomy (cf. Leung, 2004).

This paper is organized as follows: Section 2 develops the basic monocentric city model, followed by Section 3 in which the properties of the basic model are derived and it is calibrated to match a “typical” metropolitan area. We study the impact of district consolidation in Section 4.
2. Model

The structure of the model is perhaps the most basic that allows for meaningful variation in household choices. We begin with a single central workplace location and consider the location, space, and schooling decisions of households. For simplicity we have two income levels and families with two different preferences—one that favors residential space and one that favors school quality. An absentee landlord holds an auction at each location, and households bid for that location. The metropolitan area is divided into two school districts that can have different quality schools depending on the spending and peers in the schools. Education is financed through property taxes on residential land. Property taxes and school spending are determined by majority voting. Spending is determined by majority voting, and land prices are determined by household demand. Households can move costlessly between jurisdictions and choose locations that maximize their utility.

While we can prove some basic properties of the equilibrium that results, we calibrate this model to fit a stylized metropolitan area. This calibrated model permits us then to investigate how the outcomes change under governmental intervention that forces the two districts to merge. This consolidation policy matches the long history in the U.S. of combining separate school districts and reflects some recent policy proposals to have even more consolidation of small districts.

2.1. Basic structure

Imagine a city on a featureless $xy$ plane. Assume the city has all employment in a single Central Business District (CBD) located at the origin. Moreover, the city is divided into two jurisdictions, and each jurisdiction operates its own schools. The $y$-axis, passing through the CBD, forms the boundary between the two jurisdictions. We will refer to the jurisdictions as East School District (ESD) and West School District (WSD) throughout the paper. Each jurisdiction offers a local public good (education) financed through taxes on residential property.

In the labor market, the firm located in the CBD employs skilled and unskilled labor to produce a composite commodity. The equilibrium wages at the CBD are exogenous, determined by the condition that supply equals demand in the national labor market. One member of each household works and makes all the economic decisions in the house. Based on their earnings, households are categorized as skilled and unskilled worker households. Skilled workers make $w_s$ dollars per hour, while unskilled workers make $w_u$ dollars per hour. Labor is the sole source of income.

Each household has a pupil attending school, but households place different values on the quality of education a jurisdiction provides. Some value education more (high valuation types), some less (low valuation types).

---

3 An alternative formulation of city structure has been frequently applied in the urban economics literature, namely a central city with a suburban ring. This structure is more realistic in some ways than the split cities of our work, but it is less realistic in that the locational advantage of each “city” in terms of commuting is completely ordered ex-ante.

4 Note that we do not differentiate between schools and districts. It is possible that large districts have varying schools within them, but we consider just a common quality of all schools within a district.
As a result, we have four different types of households in the city $i \in \{\text{SL, SH, UL, UH}\}$, namely Skilled Low Valuation households (SL), Skilled High Valuation households (SH), Unskilled Low Valuation households (UL), and Unskilled High Valuation households (UH).

Consider a type $i \in \{\text{SL, SH, UL, UH}\}$ household seeking a residence at a location that is $r$ miles from the CBD in jurisdiction $j$. The time endowment for the household is 24 hours. The city has a dense radial transportation system. Households commute between workplace and residences. Commuting has both pecuniary and time costs. Formally, commuting requires a fixed cost of $a/2$ dollars and $b/2$ hours per mile which is evaluated at the person’s wage rate. Thus, the household’s income, net of transportation costs, is:

$$Y_i(r) = 24w_i - (a + bw_i)r$$

The preferences for households are represented by a Cobb-Douglas utility function given by:

$$U(x_i, \eta_i; q, s, z, l) = q_j^s s^g z^l$$

where $x_i + \eta_i + \gamma + \delta = 1$, $q_j$ is the quality of education in community $j \in \{\text{WSD, ESD}\}$, $s > 0$ is the lot size, $z > 0$ is the numeraire composite commodity, $l \in [0, 24]$ is leisure, $x_i \in \{x_H, x_L\}$ is the taste parameter for education and, $\eta_i \in \{\eta_H, \eta_L\}$ is the taste parameter for lot size. The normalization is done to differentiate high valuation and low valuation households with the same income.

The budget constraint of the household is given by

$$z(r) + (1 + \tau_j)R_j(r)s(r) + w_i l(r) = Y_i(r) = 24w_i - (a + bw_i)r$$

where $\tau_j$ is the property tax rate, $R_j(r)$ is the equilibrium rent per unit of land paid to a landlord for his land in community $j$. Notice that this formulation suggests that households sell all available time to employers and buy back some leisure at the prevailing market wage rate.

We can define the bid-rent function of the household, which shows the household’s willingness to pay given a fixed utility level. The bid-rent function can be mathematically expressed as

$$\Psi(r, u_i, q_j, \tau_j, w_i) = \max \left\{ \frac{Y_i(r) - z - w_i l}{(1 + \tau_j)s} \left| U(x_i, \eta_i; q, s, z, l) = u_i \right. \right\}$$

To minimize algebra, we take the duality approach (Solow, 1973) to calculate the bid-rent function and exploit the fact that with this form of utility function the optimized budget shares of lot size $s$, composite good $z$, and leisure $l$ are given by

$$\frac{\eta_i}{\eta_i + \gamma + \delta}, \frac{\gamma}{\eta_i + \gamma + \delta}, \text{ and } \frac{\delta}{\eta_i + \gamma + \delta},$$

respectively. The Marshalian demands can be calculated with these shares.

---

5 The importance of commuting costs appears throughout the theoretical literature on urban location. There is also empirical verification of its importance in a variety of places; see, for example, McMillen and Smith (2003).

6 It is implicitly assumed that each household manages the construction of his house by himself.

7 An alternative to our normalization would be to place conditions on $x/\eta$ such that a high valuation household had a greater ratio of these parameters than a low valuation person. This would permit considering individuals who valued both schools and housing high or low (with suitable adjustments in other parameters on the composite good and leisure). Any expansion to decentralized employment would, however, be much more complicated.
easily. Substituting these demands in the utility function yields the indirect utility function,

\[ V(\cdot) = \frac{k_i}{R(r)^\eta_i (1 + \tau_j)^\eta_j W_i} q_j^{\eta_j} Y_i(r)^{\eta_i + \gamma + \delta} \]

where \( k_i = \eta_i^{\eta_i / \gamma} \delta (\eta_i + \gamma + \delta)^{(\eta_i + \gamma + \delta)} \) is a constant. By the duality principle, we can derive the bid-rent and bid-max lot size functions,

\[ \Psi(r, u_i, q_j, \tau_j, w_i) = \frac{k_i^{1/\eta_i}}{(1 + \tau_j) W_i} q_j^{\eta_j / \eta_i} Y_i(r)^{\eta_i + \gamma + \delta} u_i^{1/\eta_i} \]

\[ s(r, u_i, q_j, \tau_j, w_i) = \frac{\eta_i}{(\eta_i + \gamma + \delta)(1 + \tau_j)} \frac{Y_i(r)}{\Psi(r, u_i, q_j, \tau_j)} \]

Clearly, the bid-rent function is differentiable, and decreasing in both utility level \( u \) and distance \( r \) (i.e., \( \frac{\partial \Psi(r, \cdot)}{\partial r} < 0 \) and \( \frac{\partial \Psi(u_i, \cdot)}{\partial u_i} < 0 \)). Moreover, since \( \frac{\eta_i + \gamma + \delta}{\eta_i} > 1 \), the bid-rent function is convex in \( r \). As for the bid-max lot size function, it is differentiable, and increasing in both \( u \) and \( r \). As will soon become clear, the relative steepness of bid-rent functions by distance is important in urban spatial analyses and determines the spatial ordering of equilibrium household locations in a jurisdiction. The slope of the bid-rent function \( \frac{\partial \Psi(r, \cdot)}{\partial r} \) which shows how much the household is willing to sacrifice in lot size to reside at a location one mile closer to the CBD, is given by,

\[ \frac{\partial \Psi(r, u_i, q_j, \tau_j)}{\partial r} = -\frac{\Psi(r, u_i) \eta_i + \gamma + \delta (a + bw_i)}{Y_i(r)} \]

Following Alonso (1964), we assume a competitive land market in which households bid for land and land owners offer the land to the highest bidder. For any given location, the landlord receives five implicit offers. She may rent her land to any of our four different types of households or leave the land for a non-urban purpose (e.g., agriculture). When the latter occurs, she gets a fixed bid of \( r_a \).

Consider two different types of households with bid-rent functions \( \Psi_1(r, u_1, \cdot) \) and \( \Psi_2(r, u_2, \cdot) \). In the calibrated model, we have a unique intersection point for any pair of bid-rent function, and it suffices to look at the slopes at the intersection point to determine which bid-rent function is the steepest. Let \( r^* \) stand for the intersection point (i.e., \( \Psi_1(r^*, u_1, \cdot) = \Psi_2(r^*, u_2, \cdot) \) for some \( (r^*, u_1, u_2) \)). Given that Household 1 and Household 2 are the only bidders and that Household 1 has a steeper bid-rent function (i.e., \( \frac{\partial \Psi_1(r^*, u_1)}{\partial r} > \frac{\partial \Psi_2(r^*, u_2)}{\partial r} \)), the bid-rent function of Household 1, \( \Psi_1(r, u_1, \cdot) \), dominates the bid-rent function of Household 2, \( \Psi_2(r, u_2, \cdot) \), as we move towards the CBD. In other words, the households are stratified by distance, and the equilibrium location of Household 1 is closer to the CBD than that of Household 2 if and only if the following condition holds:

\[ \frac{\partial \Psi_1(r^*, u_1)}{\partial r} \frac{\partial \Psi_2(r^*, u_2)}{\partial r} = \frac{(\eta_1 + \gamma + \delta)\eta_2}{(\eta_2 + \gamma + \delta)\eta_1} \frac{Y_2(r^*)}{Y_1(r^*)} \frac{(a + bw_1)}{(a + bw_2)} > 1 \]

\(^8\) It is the optimal lot size when we directly solve the household’s bid-rent maximization problem.
Now consider two households with the same wages, bidding for a location at distance \( r \) (i.e., \( w_1 = w_2 \) and \( Y_1(r) = Y_2(r) \)). Suppose Household 1 is a high valuation type \((x = x_H)\) and Household 2 is a low valuation type \((x = x_L)\). Since \( \eta_1 < \eta_2 \), \( \partial \Psi_1(r, u_1, \cdot) / \partial r > \partial \Psi_2(r, u_2, \cdot) / \partial r \). In other words, Household 1, who values school quality more, has a steeper bid-rent curve and locates closer to the CBD.\(^9\) Next, consider two households with the same tastes and assume Household 1 is the richer one (i.e., \( \eta_1 = \eta_2 \), \( w_1 > w_2 \)). Whether \( \partial \Psi_1(r, u_1, \cdot) / \partial r > \partial \Psi_2(r, u_2, \cdot) / \partial r \) is indeterminate. There are two opposing factors. First, higher wages imply higher incomes, and richer households want bigger houses. Thus, they want to move away from the CBD. However, because the cost of commuting is more for higher wage-earners, they want to move closer to the CBD in order to commute less.\(^10\)

We can identify the equilibrium location of households and equilibrium market rents in each jurisdiction, once we know the equilibrium wages in the CBD along with households’ utilities. As is natural within monocentric models, the results are identical along any radial from the CBD. With our discrete households, each household type will live within a ring around the CBD (but differing by school district). Market rent \( R_j(r) \) in community \( j \) is the upper envelope of the equilibrium bid-rent curves \( \Psi_i(r, u^*_i, \cdot) \) for all household types \( i \in \{\text{SL, SH, UL, UH}\} \) and the agricultural rent line (i.e., \( R_j(r) = \max\{\max_{i \in \{\text{SL,SH,UL,UH}\}} \Psi_i(r, u^*_i, \cdot), r_a\} \)). This ensures that no type \( i \) household can achieve a higher utility than \( u^*_i \), \( \forall i \in \{\text{SL,SH,UL,UH}\} \) and that farmers can make no profits. Needless to say, in equilibrium, if type \( i \) households are present in both jurisdictions, they should get the same utility wherever they are so that nobody has an incentive to switch his community of residence. Since bid-rent functions for all types are convex and decreasing in \( r \), market rent curves, \( R_j(r) j \in \{\text{WSD,ESD}\} \), are necessarily decreasing up to a distance (called fringe distance, \( \tilde{r}_j \) \( j \in \{\text{WSD,ESD}\} \)) above which the land is left for agricultural use (i.e., \( R_j(\tilde{r}_j) = r_a \) \( j \in \{\text{WSD,ESD}\} \)). Since we know the location of households and market rents in equilibrium, we can calculate lot sizes across each city in equilibrium, \( S_j(r) \).

### 2.2. Some properties of the model

This basic model immediately yields some novel outcomes that introduce more realism into the description of the equilibrium outcomes.

**Proposition 1.** Suppose \( q_{\text{ESD}} \gg q_{\text{WSD}} \) and four different types of households are bidding for land. Then, the size of the ring allocated to high (low) valuation households is larger (smaller) in the East.

---

\(^9\) Behind these equilibrium statements is a bidding process (that will become more important in the simulation exercise below). Without loss of generality, suppose that the steepness of bid-rent functions from the lowest to the highest is given as Skilled Low, Skilled High, Unskilled Low, and Unskilled High. The landlord with the knowledge of households’ willingness to pay (i.e., bid rent functions) holds an auction at each location. He knows he gets the fixed rent of \( r_a \) if he does not sell it. He gets the bid from a Skilled Low Valuation household. If it is below \( r_a \), then the spot is left for agricultural usage. If it exceeds \( r_a \), then he gets another bid from Skilled High Valuation household. If no Unskilled Low Valuation household can outbid him, the Skilled High Valuation household gets the land. Otherwise, an Unskilled High Valuation household makes an offer, and the highest bid gets the land. The order of households for bidding is defined in this way because the bidding starts at locations closer to the CBD and moves outwards, and the landlord knows he can get the highest (lowest) bid from an Unskilled High (Skilled Low) Valuation household.

\(^{10}\) de Bartolome and Ross (2003) concentrate on the commuting cost motivation but do not allow for differing lot sizes or housing quality.
Proof 1. Consider a Skilled High Valuation household in the East School District. The bid-rent function for Skilled High Valuation household, $W_{SH}(r, u_{SH}, \cdot)$, dominates that for Unskilled Low Valuation households, $W_{UL}(r, u_{UL}, \cdot)$, if $r \geq r_{ESD}$, where $r_{ESD}$ is the intersection distance such that $W_{SH}(r_{ESD}, u_{SH}, q_{ESD}, \tau_{ESD}, \cdot) = W_{UL}(r_{ESD}, u_{UL}, q_{ESD}, \tau_{ESD}, \cdot)$. Because identical households achieve the same utility level regardless of location, notice that

$$W_i(\cdot, q_{WSD}, \tau_{WSD}) = \left(1 + \tau_{ESD} \frac{q_{WSD}}{\eta_i} \right) \frac{q_{WSD}}{q_{ESD}} W_i(\cdot, q_{ESD}, \tau_{ESD}) \quad \forall i \in \{SL, SH, UL, UH\}$$

And note that at distance $r_{ESD}$ in the West School District,

$$\frac{W_{SH}(\tilde{r}_{WSD}, u_{SH}, q_{WSD}, \tau_{WSD})}{W_{UL}(\tilde{r}_{WSD}, u_{UL}, q_{WSD}, \tau_{WSD})} = \left(\frac{q_{WSD}}{q_{ESD}}\right)^{\alpha_H/\eta_H - \alpha_L/\eta_L} \ll 1 \text{ if } \alpha_H/\eta_H - \alpha_L/\eta_L > 0$$

Since bid-rent functions are convex and differentiable, it follows that $r_{ESD} \ll r_{WSD}$ where $W_{SH}(\tilde{r}_{WSD}, u_{SH}, q_{WSD}, \tau_{WSD}, \cdot) = W_{UL}(\tilde{r}_{WSD}, u_{UL}, q_{WSD}, \tau_{WSD}, \cdot)$; that is, the Unskilled Low Valuation households outbid Skilled High Valuation households on a larger semi-circular piece of land in the West. By the same logic, one can conclude that, in the West, the Skilled Low Valuation households outbid Skilled High Valuation households on a larger semi-circular piece of land. Compared to an Unskilled High Valuation household, the piece of land at which Skilled High Valuation households outbid Unskilled High Valuation households would be the same as that in the East. Similarly, one could show the proof is the same for all other types, $\forall i \in \{SL, UL, UH\}$.

Now, let us introduce the fifth alternative, the agricultural use of land. Theoretically, a variety of outcomes are possible, but the following proposition holds. □

Proposition 2. Suppose $q_{ESD} \gg q_{WSD}$ and the landlord holds an auction with five alternative bids. Then, it cannot be case that there is perfect stratification by income. At least, one community should be heterogenous in income, as opposed to the traditional Tiebout models.\footnote{As noted, Epple and Platt (1998) develop a Tiebout model with mixed income households through the introduction of varying preferences. They do not, however, have the joint effects of employment access and concentrate on local income redistribution. de Bartolome and Ross (2003) develop a model of fiscal competition with the possibility for income mixing that has some features similar to the model here but that will not permit school quality considerations such as those below. Households purchase a fixed lot, and location comes from the trade-off of commuting costs, land prices, and a voter determined tax bill (which equates to public good quality).}

Proof 2. Without loss of generality, assume that the steepness of bid-rent functions from the lowest to the highest would be Skilled Low, Skilled High, Unskilled Low, and Unskilled High Valuation households. If it is the case that households with the flattest bid-rent function (Skilled Low Valuation types) happen to live in the West School District, then by Proposition 1, it must be also the case that Unskilled Low Valuation households are present in the West School District. If some Skilled Low Valuation types were to live in the East School District, again by Proposition 1, Skilled and Unskilled High Valuation households should also be present in the East School District. □
As is common, we assume our city is closed (i.e., the population of each of our four types of households are exogenously given) and the form of ownership is absentee ownership in which the land is owned by absentee landlords. Let $\bar{N}_{SL}, \bar{N}_{SH}, \bar{N}_{UL},$ and $\bar{N}_{UH}$ stand for the population of Skilled Low Valuation Households, Skilled High Valuation households, Unskilled Low Valuation households, and Unskilled High Valuation households, respectively.

We denote the land density at distance $r$ by $L(r)$. By definition, the amount of land available for housing between the distance $r$ and $r + dr$ is $L(r)dr$. Since we have radial symmetry around the CBD, $L(r)$ is simply $L(r) = \pi r$ in either jurisdiction. Also, let $n_j(r)$ be the equilibrium density function of the household distribution in jurisdiction $j \in \{\text{WSD, ESD}\}$. That is, the number of households between the distance $r$ and $r + dr$ equals $n_j(r)dr$ in jurisdiction $j$. Without loss of generality, let the equilibrium residence of a location at distance $r$ in jurisdiction $j$ be a type $i$ household. Then, the marginal household population at distance $r$ in community $j$ is given as $n_j(r) = \frac{L(r)}{S_{WSD}(r)}$. Also, since $\bar{N}_i, i \in \{\text{SL, SH, UL, UH}\}$, type $i$ households reside in the city, the population constraint for type $i$ households is stated as follows:

$$\int_0^\infty \frac{L(r)}{S_{WSD}(r)} I[t_{WSD}^i(r) = i]dr + \int_0^\infty \frac{L(r)}{S_{ESD}(r)} I[t_{ESD}^i(r) = i]dr = \bar{N}_i$$

where $t_j^i(r), j \in \{\text{WSD, ESD}\}$, is a function showing the occupant of the location at distance $r$ in jurisdiction $j$ when in equilibrium. The indicator function, $I[t_j^i(r) = i], j \in \{\text{WSD, ESD}\}$, takes the value 1 if the equilibrium resident of location at distance $r$ in jurisdiction $j$ is a type $i$ household, 0 otherwise. The population constraint implicitly assumes that the land market clears in jurisdiction $j \in \{\text{WSD, ESD}\}$. Formally,

$$S_j(r)n_j(r) = L(r) \quad \forall r \leq \tilde{r}_j$$

We can also calculate the household density at distance $r$ in jurisdiction $j$, $\rho_j(r)$ (i.e., the number of households per unit of land at distance $r$ in jurisdiction $j$). Then, by definition

$$\rho_j(r) = \frac{n_j(r)}{L(r)} \quad j \in \{\text{WSD, ESD}\}$$

Since the lot size curve is increasing in $r$, the household density curve is decreasing in $r$ up to the fringe distance in both jurisdictions.

2.3. Taxes and school quality

From a household’s point of view, the public goods aspect of each jurisdiction is characterized by the quality of education and property tax rate pair $(q_j, \tau_j)$ it provides. Now, we turn to the question of determining $(q_j, \tau_j)$ in community $j \in \{\text{WSD, ESD}\}$. Education is financed through property taxes on residential land. Each jurisdiction’s local government spends all tax revenue on education. Then, the government budget constraint in community $j$ is

$$e_j = \tau_j \tilde{R}_j = \tau_j \int_0^{\tilde{r}_j} \frac{R_j(r)L(r)}{N_j} dr$$
where $N_j$ is the population, $e_j$ is the expenditure per pupil, and $R_j$ is the tax base per pupil in community $j$. Characterizing the quality of education has historically proved difficult. The educational production function literature has not provided a clear picture of how to specify the underlying elements of quality (see Hanushek, 2003). Here we pursue the peer aspects, an element frequently identified in the residential location literature. The quality of education in community $j$ is specified by a production function

$$q_j = \pi_j(N_{Sj}, N_{Uj})e_j$$

where $N_{Sj}$ and $N_{Uj}$ are the number of skilled and unskilled worker households in community $j$, respectively. The peer group effect function (efficiency) $\pi_j(\cdot)$ is given by

$$\pi_j(N_{Sj}, N_{Uj}) = c_1 + \frac{N_{Sj}}{c_2N_{Sj} + c_3N_{Uj}}$$

where $c_1, c_2, c_3 > 0$ are constants. Notice that the efficiency of schools in jurisdiction $j$, $\pi_j$, is increasing in skilled worker households and decreasing in unskilled worker households (i.e., $\frac{\partial \pi_j(\cdot)}{\partial N_{Sj}} > 0$ and $\frac{\partial \pi_j(\cdot)}{\partial N_{Uj}} < 0$). Two arguments can be made to justify this kind of peer group effect. The first argument is based on the classical peer-group learning effect: the more my neighbor knows, the more I can learn from him. The second argument is that skilled workers are more involved in how schools operate such as taking a part in the schooling process as board members. This involvement may, for example, lead to a more efficient use of resources.

The property taxes are determined by majority voting in each jurisdiction. Following Epple et al. (1984, 1993), we assume that voters are myopic in the sense that they do not consider that their decision about $(q_j, \tau_j)$ will influence land prices, populations, and efficiency in the schooling systems. Then, a type $i$ household at distance $r$ in community $j$'s preferred tax rate is given by the following problem:

$$\max_{\tau_j} V(\cdot) = \frac{k_i}{R(r)^\eta(1 + \tau_j)^\nu} q_j^\alpha Y_j(r)^\eta(1 + \delta) \quad \text{subject to} \quad q_j = \pi_j(\cdot)e_j$$

Solving this problem yields the preferred tax rate for type $i$ household, $\bar{\tau}_i = \frac{s_i}{\eta - \nu}$. It is worth pointing out that the preferred tax rate is independent of income and is a function of the household’s valuation type. Since there are only two valuation types for households, there are two possible preferred tax rates in the economy, and high valuation types have a higher preferred tax rate ($\bar{\tau}_{SH} > \bar{\tau}_{SL}$ and $\bar{\tau}_{UH} > \bar{\tau}_{UL}$). Also, the higher they value housing and spend on it (higher $\eta$), the lower the property tax rate they prefer.

The timing of events would be as follows: at the beginning of each period, households make community/residential choice decisions with the expectation that the last period’s education and property tax packages would prevail in the current period. Once they move in, they are stuck. They vote for the property tax rate in their community of residence, and the public good and tax rate package might be different from what they expected. Since they are temporarily immobile, they do not have much choice but to consume what the
community offers. At the beginning of the next period, they update their expectations and events start over again.

3. Equilibrium

**Definition 1.** An equilibrium is a set of utility levels \( u_i^* \) \( \forall i \in \{SL, SH, UL, UH\} \), market rent curves \( R_j(r) j \in \{w, e\} \), quality of education and property tax pairs \( (q_j, \tau_j) j \in \{WSD, ESD\} \), household population distribution functions \( n_j(r) j \in \{WSD, ESD\} \), and type functions \( t_j^* r \in \{WSD, ESD\} \) which show the equilibrium occupant of the location at distance \( r \) in community \( j \) such that:

- Different household types bid for each location. The land at a location is developed for the highest bidder if the highest bid exceeds the fixed non-urban purpose bid of \( r_d \). Otherwise, it is not developed.
- All job opportunities are offered by a firm located at the CBD. Wages in the CBD are exogenously determined. The city has a dense radial commuting system. Households commute to workplaces. Commuting has both pecuniary and time costs.
- Regardless of their location or communities, households of the same type attain the same utility level (i.e., a type \( i \) household gets \( u_i^* \) anywhere, in equilibrium).
- The metropolitan area contains two jurisdictions, each of which operates its own schools. Moreover, it is a closed area (i.e., populations for each type are exogenous) and the land is owned by absentee landlords.
- The local public good, education, is produced through a production function defined by peer characteristics and school spending, where spending is financed through local property taxes on residential land as determined by majority voting in each jurisdiction.
- Labor and land markets clear.
- The local government budget balances in all jurisdictions.

3.1. Calibration

Since we are interested in empirical implementation, further development of the model can be best done in a more fully parametrized context. Our calibration is based on aggregate data for a “typical” U.S. metropolitan area with parameters given in Table 1.

Recall that the household spends \( n_H \), \( \gamma_H \), and \( \delta_H \) percent of his net, after tax income \( Y_H(r) \) on land, composite commodity, and leisure, respectively. In the U.S., average weekly hours of persons working full time is about 40 hours,\(^{15}\) and the average annual earnings of 18-year or over high school and college graduate workers are $22,154 and $38,112, respectively, in 1997. These figures suggest the hourly wages in the CBD for unskilled and skilled workers should be calibrated as \( w_u \approx \$10.7/\text{hour} \) and \( w_s \approx \$18.3/\text{hour} \), respectively. The share of leisure in the household’s budget is

\[
\frac{\delta}{n_H + \gamma + \delta} = 1 - \frac{40w_u}{24 \times 7 \times w_s} \approx 0.762.
\]

While housing expenditures vary across U.S. metropolitan

\(^{15}\) The statistical facts, unless otherwise indicated, come from U.S. Bureau of the Census (1998).
areas, we assume that a household spends about 20% of his income on shelter. Therefore, we set the budget share of composite commodity and land as
\[ c + H + g H + c + d = \frac{1 - 0.762}{C_0} \times 0.1904 \text{, respectively.} \]
Recall that the preferred tax rate for a type i household is given by
\[ \tilde{\tau}_i = \frac{a_i g_i}{C_0} \] and we have two possible preferred tax rates, one for high valuation and another for low valuation type households. The one for high (low) valuation type is set to be about 1.7% (1.3%). From this, we have sufficient information to calibrate \( a_H, a_L, g_H, g_L, c, \) and \( d. \)

Pecuniary commuting cost per round trip mile is based on the cost of owning and operating an automobile. In 1997, pecuniary cost per mile was 53.08 cents, suggesting a pecuniary commuting cost of \( a = \$1.1 \) per round trip mile. Assuming the commuting speed is 20 miles/h within the city, the time cost of commuting per round trip mile is set to be \( b = 0.1 \) hours (Table 1).

In equilibrium, the endogenous urban fringe distance is calibrated to be about 10 miles in both jurisdictions. The population of the city is set to be 1,469,748 households, which implies approximately a population density of 4680 households per square mile.16 Approximately, 40% of the total population is assumed to be skilled worker households. Moreover, 30% of skilled households are assumed to be low valuation type. As for the unskilled households, 70% are low valuation types. (See Table 2 below for the exact decomposition.) The agricultural rent bid 17 \( r_a \) is set to be \( \$14,828 \) per acre per year.

The pairs of \((q_j, \tau_j)\) \( j \in \{\text{WSD, ESD}\} \) which are consistent with the population distribution summarized in Table 2 are found. Parameters of the education production function are set to be \( c_1 = 0.2, c_2 = 0.5778, c_3 = 0.5521 \) so that \((q_j, \tau_j)\) preferences of households in jurisdiction \( j \in \{\text{WSD, ESD}\} \) are consistent with \((q_j, \tau_j)\) pairs underlying the population distribution. We employ discrete distances and evaluate integrals numerically.

From the urban economics literature, we know that, if we had only one jurisdiction in the model (i.e., both jurisdictions provide the same quality of education and have the same tax

---

16 The median population per square mile of cities with 200,000 or more population was 3546 in 1992. Source: U.S. Bureau of the Census (1994).

17 The agricultural bid rent is assumed to be constant, independent of location since agricultural activity does not play an important role in urban land use theory.
rates), we could theoretically show the existence and uniqueness of equilibrium by using boundary rent curves approach.\(^\text{18}\) Since we have two jurisdictions, it is difficult to show the uniqueness analytically. However, since \(\frac{\partial u}{\partial \gamma_L} - \frac{\partial u}{\partial \gamma_H} > 0\), from Proposition 1 we expect an equilibrium with at least one heterogenous communities in income with Skilled and Unskilled High (Low) Valuation types present in the East (West). Moreover, with the current set of calibration parameters, a numerical search suggests that we have a unique equilibrium.

3.2. Benchmark equilibrium

The equilibrium is summarized by Tables 2 and 3 along with Figs. 1 and 2. The distribution of households across jurisdictions is shown in Table 2. The East School District attracts higher proportions of families that value education highly, while the West School District is home for families with lower average valuation of schools. The West School District also tends to have higher concentrations of poor people, but districts have a much more even income distribution than school valuation distribution. The East School District contains 52.4\% of the household population.

Table 3 indicates that skilled worker households in the benchmark equilibrium attain a higher level of utility than unskilled worker households, owing to the fact that they earn more. And, in our basic calibration, high valuation type households also obtain a higher utility than low valuation types.

The distribution of households in equilibrium reflects the schools and fiscal position of the districts.\(^\text{19}\) The East School District is more efficient than the West School District, because it attracts a better peer group (i.e., relatively more skilled households).\(^\text{20}\) The East School District ultimately offers a better education, because it combines more efficient schools with higher spending (i.e., higher property taxes that yield higher expenditure per pupil).\(^\text{21}\)

Table 2
The distribution of households across jurisdictions

<table>
<thead>
<tr>
<th>Types</th>
<th>West (%)</th>
<th>East (%)</th>
<th>Total (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skilled low</td>
<td>11.9</td>
<td>0.2</td>
<td>12.1</td>
</tr>
<tr>
<td>Skilled high</td>
<td>5.7</td>
<td>22.2</td>
<td>27.9</td>
</tr>
<tr>
<td>Unskilled low</td>
<td>28.1</td>
<td>14.8</td>
<td>42.9</td>
</tr>
<tr>
<td>Unskilled high</td>
<td>1.9</td>
<td>15.2</td>
<td>17.1</td>
</tr>
<tr>
<td>Total</td>
<td>47.6</td>
<td>52.4</td>
<td>100</td>
</tr>
</tbody>
</table>

---

\(^\text{18}\) See Fujita (1989) for the formal procedure.

\(^\text{19}\) Ex-ante both jurisdictions are identical. Hence, if we re-label West as East and East as West, we obtain a symmetrical solution. Depending on the initial point, West is sometimes the efficient district. Recall, for the moment, the model is dynamic and what we have is the stationary equilibrium of that model. Which equilibrium we end up with is a matter of the public good and property tax packages, \((q_j, \tau_j) j \in \{WSD, ESD\}\), that each jurisdiction starts with and offers over time. This actually means an inefficient jurisdiction \(j\) could be the efficient jurisdiction by changing the \((q_j, \tau_j)\) package for the current period. The market handles the rest.

\(^\text{20}\) If peers are defined in terms of how families value education, similar results are obtained in terms of mixing of households.

\(^\text{21}\) In 1997, the average expenditure per pupil was $5923 of which 45 percent comes from local funds. Source: U.S. Department of Education (2004).
cross-sectional view of the city in equilibrium. The West School District is located on the negative $x$-axis, and the CBD is located at the origin. Skilled worker households choose to locate away from the CBD. The inner rings around the CBD are occupied by unskilled worker households. With the current set of parameters, skilled worker households have a higher income and want to have bigger houses. The commuting cost factors are dominated by the income effect. They move away from the CBD where such houses are more abundant and cheaper. In the taste dimension, the households who value schooling less have a higher income elasticity of lot size ($\eta_L > \eta_H$). Thus, they enjoy bigger houses which are more abundant and cheaper as they move away from the CBD. The spatial equilibrium ordering of households are given as Unskilled High Valuation households (UH), Unskilled Low Valuation households (UL), Skilled High Valuation households (SH), Skilled Low Valuation households (SL), ordered from the CBD to fringe.

We see market rents ($\text{per square foot}$) go down as we move away from the CBD; locations closer to the CBD have higher accessibility advantages/disadvantages that are...
capitalized into rents. More importantly, we see a jump in the market rent as well as higher average gross market rent as we cross to the East School District from the West School District due to the capitalization of quality of education difference. Fig. 2 shows lot sizes in square yards at the equilibrium. Not surprisingly, households have bigger houses as we move away from the CBD.

The important aspect of this simple model of household location is that it captures some of the complexity of urban areas, where complete stratification is not the observed outcome. Indeed, because households choose locations in terms of a complicated set of factors, similar households in terms of both incomes and tastes end up living in different jurisdictions. Even without introducing complications due to dispersed employment locations, we find that individuals make trade-offs that are neglected in standard models but that are important for assessing major policy changes.

4. School district consolidation

This stylized model provides a vehicle for assessing the key trade-offs in a variety of policy options. Specifically, states frequently change their financing of local schools and change the institutional rules. Most of the policy discussions around these changes tends to presume that households do not react to changes. But, in fact, households will alter their behavior. This section applies the previous model to the evaluation of state policies related to district structure.

One of the clearest cases of state actions involves the consolidation of school districts. The twentieth century marked the most dramatic change in U.S. education system. Over 100,000 school districts have been eliminated through consolidation since 1938, a drop of almost 90% (U.S. Department of Education, 2004). While the pace of school district consolidation has slowed since the early 1970s, some states still provide incentives to consolidate.

Although the causes of consolidation (Brasington, 1999) and the cost savings from school district consolidation (Duncombe and Yinger, 2005) are well documented, the potential efficiency consequences of consolidation has been ignored. In his seminal work, Tiebout (1956) argued that local public goods could be provided efficiently through household sorting across alternative jurisdictions. Later, Hamilton (1975) showed that Tiebout’s system with zoning provides for an efficient level of public good such as education. In the Tiebout–Hamilton world, the property tax becomes essentially a fee for services and has no deadweight lost. One would then expect, on efficiency ground, that the elimination

<table>
<thead>
<tr>
<th>Variable</th>
<th>West</th>
<th>East</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality of education</td>
<td>4.55</td>
<td>6.05</td>
</tr>
<tr>
<td>Expenditure per pupil per year</td>
<td>$1945</td>
<td>$2293</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.854</td>
<td>0.964</td>
</tr>
<tr>
<td>Tax rate</td>
<td>1.3%</td>
<td>1.66%</td>
</tr>
<tr>
<td>Average monthly gross rent per square foot</td>
<td>$0.0748</td>
<td>$0.0827</td>
</tr>
<tr>
<td>Utility level</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Skilled household</td>
<td>13.45</td>
<td>13.20</td>
</tr>
<tr>
<td>Unskilled household</td>
<td>11.76</td>
<td>11.50</td>
</tr>
</tbody>
</table>
of Tiebout alternatives as a result of school consolidation would have some efficiency cost. To see the impact, we consolidated school districts in our monocentric employment framework.

We assume the economy is at the equilibrium described at the benchmark and the government steps in to consolidate school districts (i.e., moving the metropolitan area to single school, operated by the government).\textsuperscript{22}

The new equilibrium is summarized in Table 4. The presence of peer group effects increases the inertia of moving from the benchmark to the new equilibrium. High valuation households are likely to stick together to get a better education in the short run. In the long run, we do not see segregation due to the presence of peer group effects.\textsuperscript{23}

It is most interesting to compare the new equilibrium with the benchmark case in Table 3. Both jurisdictions now provide the same quality of education and charge the same property tax rates, making the East School District a replica of the West School District. The West School District now has a higher efficiency due to the increase in the high valuation household population, but this is offset by lower efficiency in the East. Since, in terms of overall population, low valuation types are a majority, not surprisingly the tax rate prevailing in the equilibrium is that of a low valuation household. High valuation households facing higher rent and a worse education level, tend to move to the West School District where the land is cheaper. This move pushes up the rents in the West School District and pulls the rents down in the East School District. In the new equilibrium, although high valuation households prefer a higher property tax rate to boost the expenditure on education, their voice cannot be heard. Therefore, they are worse off. As for low valuation households, they are worse off as well due to the rent increase caused by the mobility of the East School District residents. Everybody suffers in that, at least by our welfare measure defined as the normalized sum of household utilities, welfare falls.\textsuperscript{24}

\begin{table}
\centering
\begin{tabular}{|l|c|c|}
\hline
Variable & West & East \\
\hline
Quality of education & 4.89 & 4.89 \\
Expenditure per pupil per year & $1956 & $1956 \\
Efficiency & 0.912 & 0.912 \\
Tax rate & 1.3\% & 1.3\% \\
Average monthly gross rent per square foot & $0.076 & $0.076 \\
\hline
\end{tabular}
\caption{Equilibrium characteristics of consolidated school districts}
\end{table}

\textsuperscript{22} As noted previously, we do not consider the possibility that school quality differs within each district. Past empirical evidence suggests that there are significant differences in quality within districts, but it is not clear that these differences are stable over time or in the face of alternative policies. Because the most significant aspect of school quality is teacher quality, alternative assignment of teachers could lead to significant changes in school quality (see Hanushek et al., 2006). All of these issues are, however, beyond the scope of this paper.

\textsuperscript{23} Nechyba (2000, 2003) introduces heterogeneity of households into his simulations, but his equilibrium results heterogeneity over time is assured by the structure of the simulations.

\textsuperscript{24} We did not report cardinal measures of loss or gain due to the fact that it varies with any monotone transformation of utility functions, although equilibrium remains the same. It is also possible that different social welfare functions could produce other outcomes. It is possible, for example, that government has purely distributional preferences that enter into the social welfare function, but such a social welfare function would not be consistent with citizen preferences.
It is noteworthy that the exposure index in terms of the skill distribution in the West goes up to 40.03%, indicating more peer interaction. With peer effects that induce externalities in location, it is not obvious how welfare is affected by government intervention. Here, we could infer that the government consolidated school districts in order to provide more equal educational opportunity to the poor. Yet, due to distortions arising from general equilibrium adjustments, every household—including the poor—get hurt.

It is surprising to see that all households are hurt with consolidation because of the magnitude of district consolidation that occurred in the United States during the twentieth century. This result is, however, consistent with the arguments of Fischel (2006), which describe consolidation as a policy externally imposed on districts and not a policy that residents voluntarily choose.

5. Conclusions

We present a residential choice model that unifies two artificially separated streams of literature in urban public finance. Each of the separate “Alonso” and “Tiebout” strands of the literature have unrealistic outcomes that lead to questions about how to interpret any policy experiments grounded in a particular viewpoint. Together, however, these approaches complement each other. Our combined model successfully explains the heterogeneous spatial location patterns we observe in the U.S. Contrary to traditional models, households are no longer stratified into communities by their income and tastes. Jurisdictions contain households with mixtures of income and taste.

A key element of our joint consideration of urban location and Tiebout sorting is that households face a trade-off between different aspects of their location. The integration of this locational aspect vividly demonstrates how partial analyses concentrating on just one can yield a distorted picture of the equilibrium resulting from simple policy choices.

This model is employed to illustrate the impacts of school district consolidation, a popular policy of the last century that interacts with the multiple locational incentives. The various states in the U.S. have pursued an active policy of consolidating smaller districts in order to make larger, “more efficient” districts. Such policies, however, do not occur in a vacuum, and households will respond to them. Moreover, the general equilibrium outcome does not hold all features of location constant—a situation seldom considered in policy deliberations. Our evaluation, based on the simple model of conflicting locational incentives, introduces some question about the overall impact of these policies.

This analysis confirms the importance of general equilibrium considerations when governments attempt policy changes that dramatically alter individual market preferences. The locational and fiscal changes that result from new policy can result in very different outcomes that are different from the naive partial equilibrium view.

Acknowledgments

We are grateful to the comments and suggestions of Dennis Epple, Ken Judd, Charles Leung, Lance Lochner, Michael Wolkoff, Yi Jin, and the seminar participants at the Federal Reserve Bank of Philadelphia, Econometric Society North American Summer Meeting, and several conferences. This research was funded by grants from the Spencer Foundation and the Packard Humanities Institute.
References


